Statistical kinks in the GOLF cross spectra, an hypothetical link toward the g-mode detection.

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Several flavours of an analysis using cross spectra, but still far to be at a conclusive level.

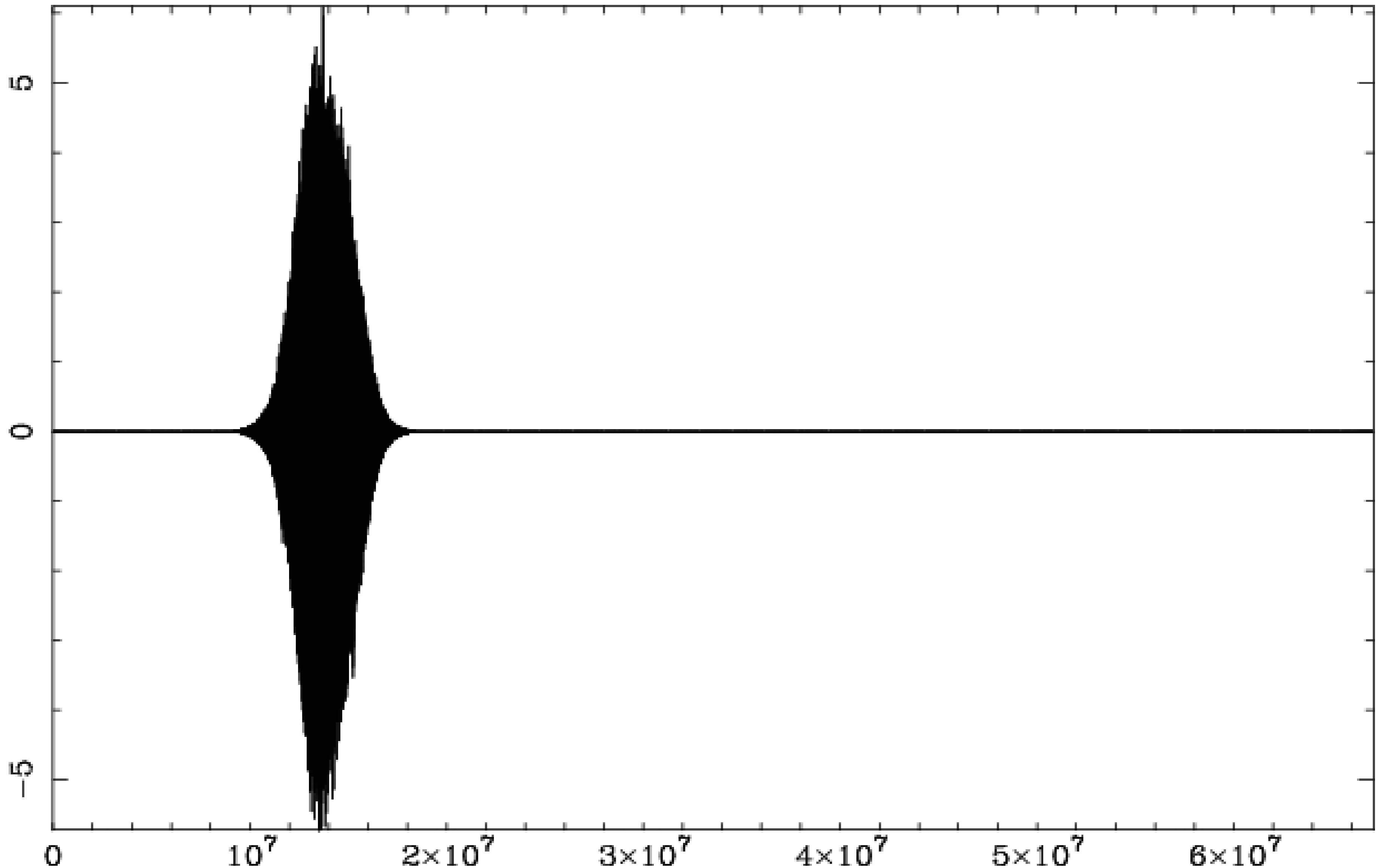
Method :

10 yr of velocity signal are separated on successive Gaussian windows (7 months long for the analysis shown below).

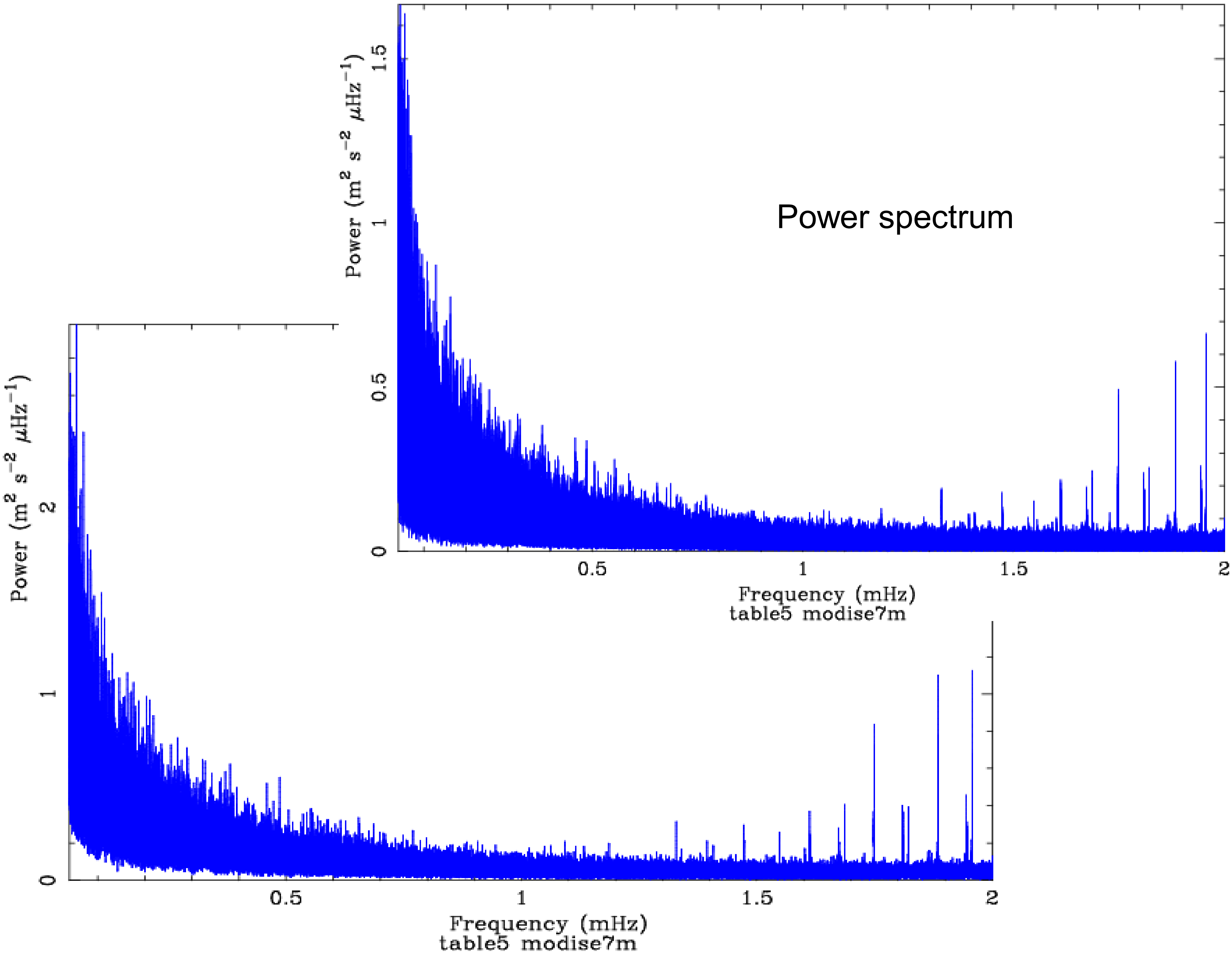
**The oversampled FFT are calculated without time shift of the data.
This calculation gives the minimum level of artefacts as side lobs or sampling effect in the frequency detection. The phase of the signal is not perturbed.**

**We can then calculate a series of cross spectrum (CS), as the product of the FFT in different windows, taken 2 by 2.
That is an extension of the power spectrum (PS), squared FFT in a single window.**

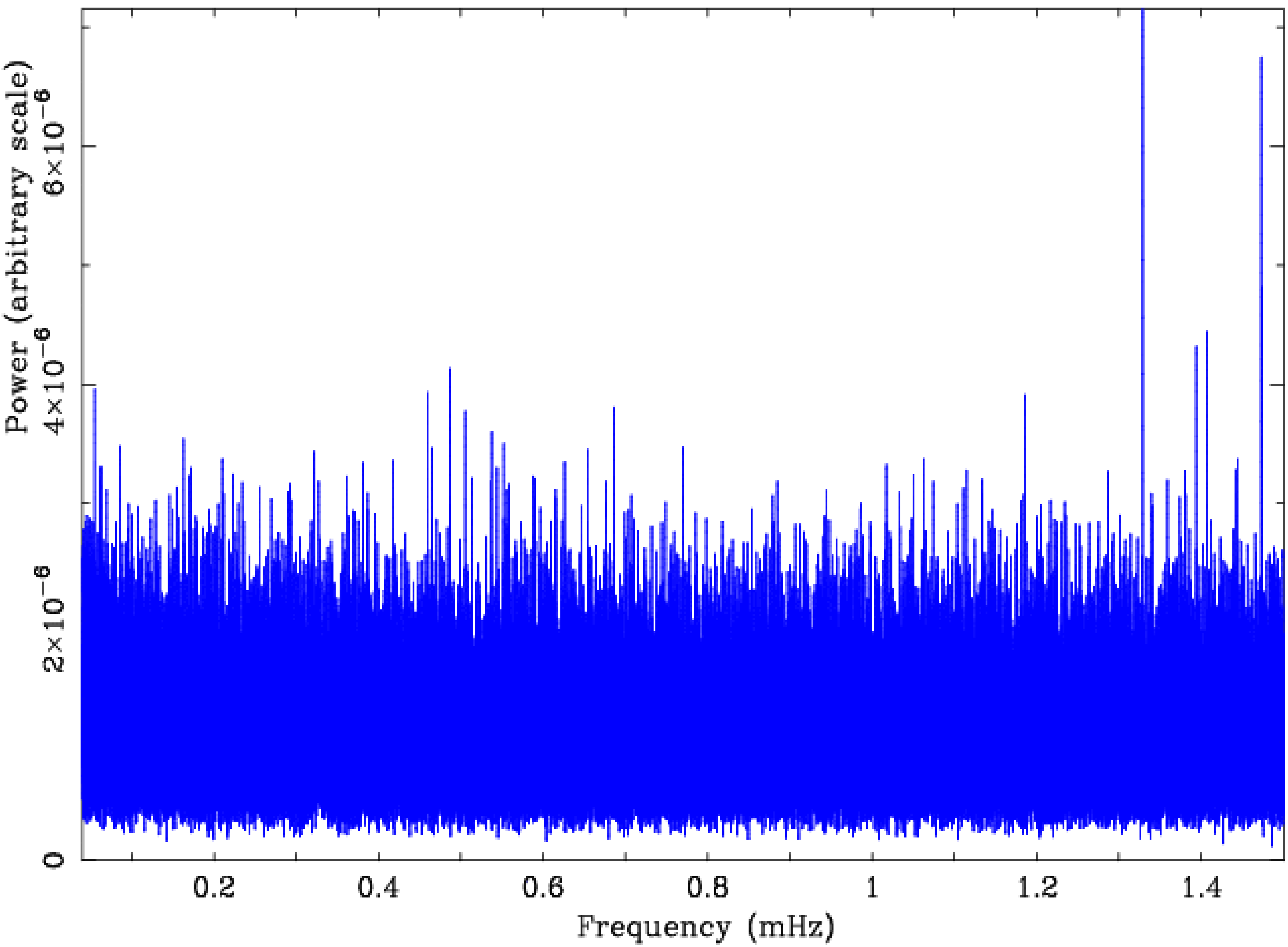
The CS are complex functions, when averaged in modulus the result is scaled in energy and similar to a PS.



10 yr of velocity signal are separated on successive Gaussian windows. The oversampled FFT are calculated without time shift.



We calculate the averaged amplitude of the cross spectrum, compared here to the averaged power spectrum (top)



The averaged cross-spectrum is normalized, a running average is used to remove the low frequency trend, dividing each bin by the sliding average. The solar background becomes a flat energy signal.

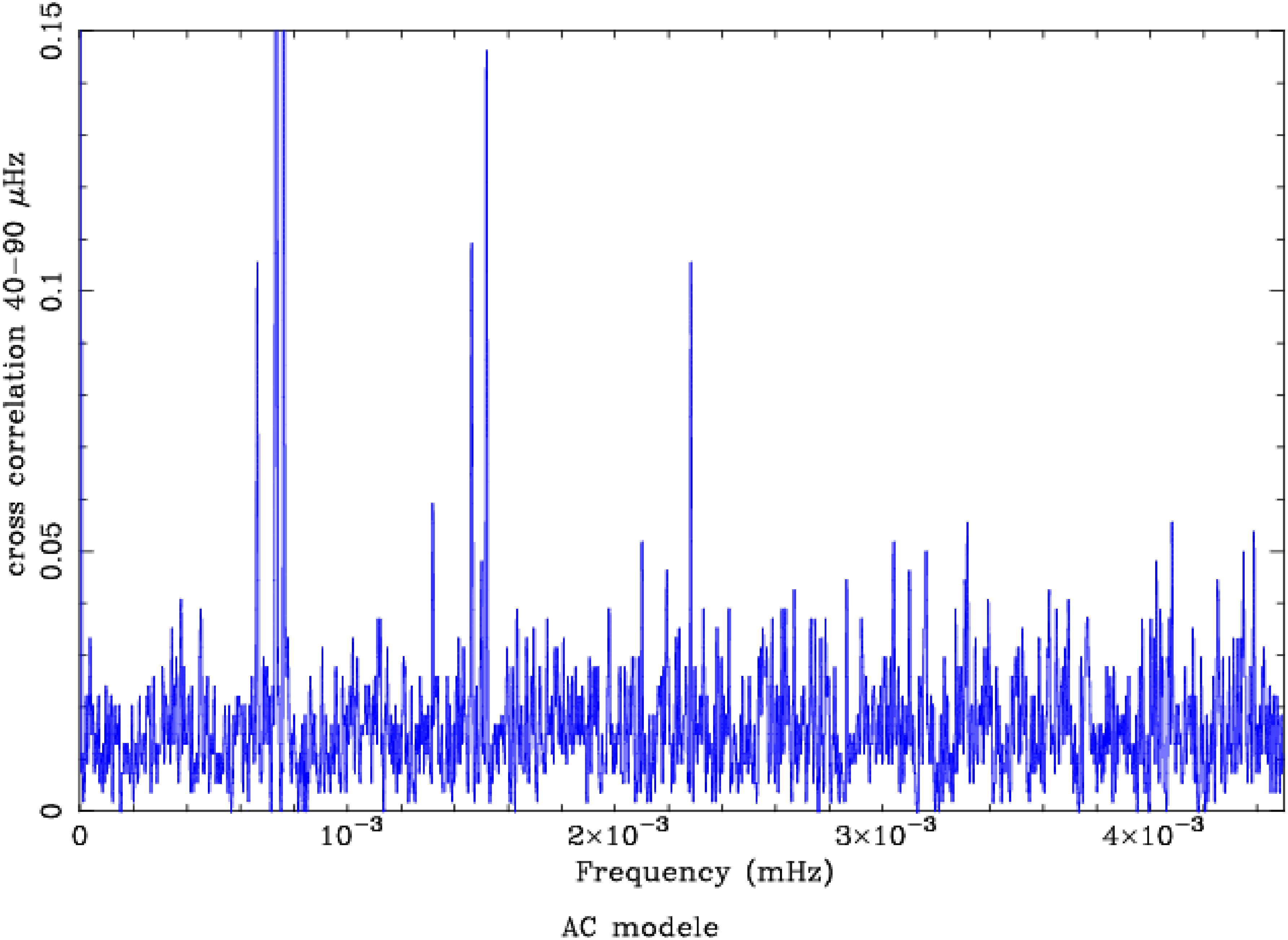
Part 1 - Search for a global signature of the splitting:

Method:

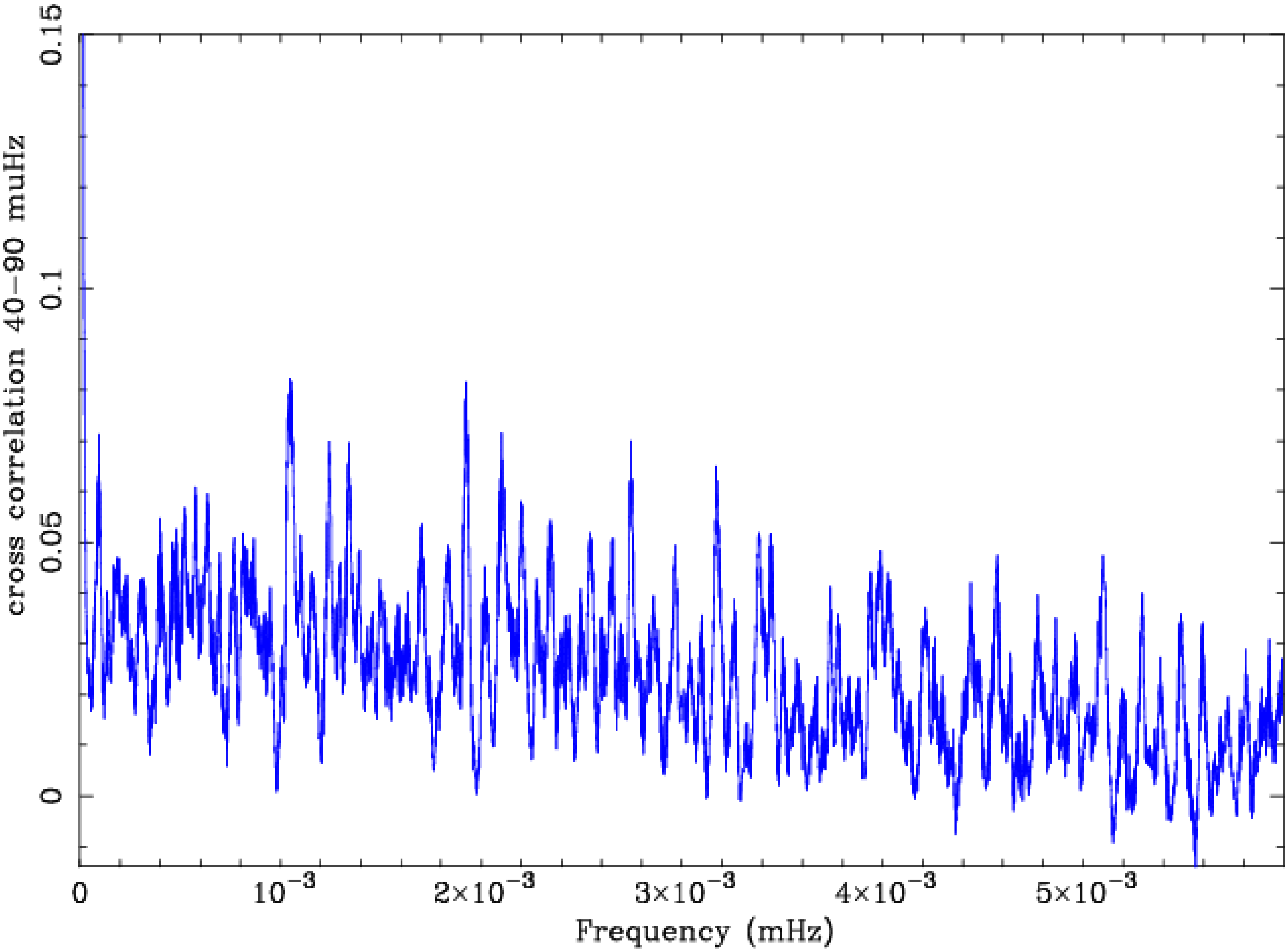
In the region where the g-mode density is relatively hight, we calculate the autocorrelation of the normalized cross spectrum.
In fact, the calculation is made from 40 µHz to 90 µHz.
It shows a 1 µHz relative maximum of the correlation.

The same function is calculated for a noiseless spectrum deduced from the numerical tables l =1 to l =4, with the sinodical value calculated fro the splitting.
It shows a 0.8 µHz relative maximum of the correlation, corresponding to the tabulated splitting for l > 1.

The 2 AC are similar for the number of replica.
So, let assume this result significant and take 1 µHz as the value of the splitting.
It corresponds to a core rotating faster than the surface.



Autocorrelation of the numerical model from 40 to 90 μHz



Autocorrelation of the above cross spectrum from 40 to 90 μHz

Part 2 - Use of the phase information in the cross spectra :

A cross spectrum is a complex function, for a given frequency, the phase is randomly distributed over 2 π in the complex plane if there is no phase coherence between the windows. It is the case for a purely random signal.

We select the bin having an “almost" real value with the test:

- real part > 0
- imaginary part / real part < φ, φ = 0.01, ~ 0.5 degree.

This test is made for every frequency bin, when the signal is phase coherent, we record the frequency bin for witch we obtain a real value. This test corresponds to a phase-coherent signal.

At the end, we have a function $N_W(\nu)$ where N is the number of “real” bins in the cross-spectra for a set of windows of length W and for the frequency ν.

The detection function $D_W(n)$ is the number of frequency bins for which $N_W(\nu) = n$

For a given value of W, we can compare the detection function D (n) with a probability function P (n) assuming all detections are randomly distributed:

$$P_W(n) = (P_W(1) / B_0)^n \quad \text{where } B_0 \text{ is the total number of bins of the FFT.}$$

For a given value of W, we can compare the detection function D (n) with a probability function P(n) calculated with the assumption of random distribution of the phase in the cross spectra :

$$R(n)= (P(1) / P(0))^{n} \quad \text{where obviously } B_0 = P(0)$$

and $$P(n)= (R(n) * P(0))$$

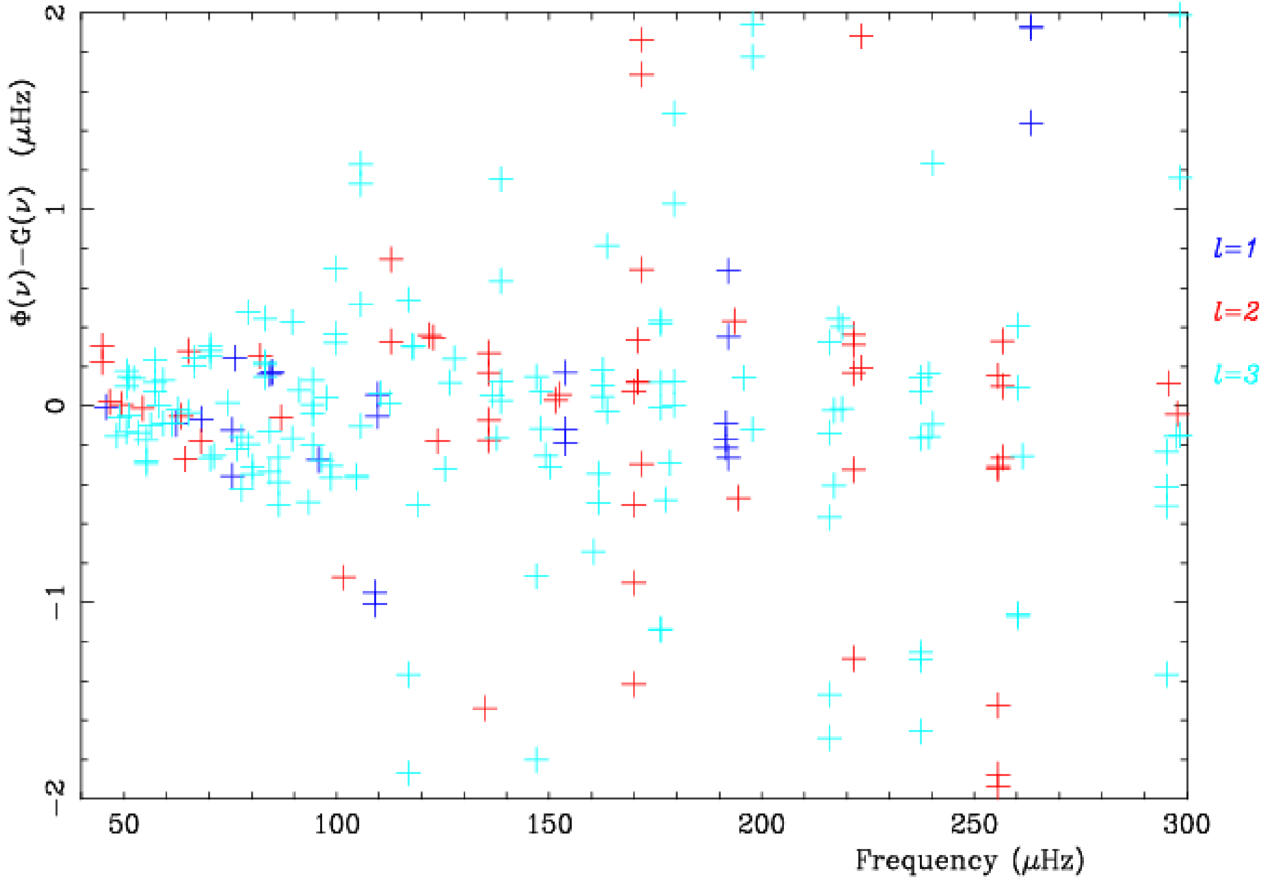
example for W = 240 d (8 months)

n	D(n)	D(n)/D(0)	R(n)	P(n)
0	476474	0.100E+01	0.100E+01	476474
1	50379	0.106E+00	0.106E+00	50379
2	5023	0.105E-01	0.112E-01	5326
3	793	0.166E-02	0.118E-02	563
4	92	0.193E-03	0.125E-03	59
5	17	0.357E-04	0.132E-04	6
6	3	0.630E-05	0.140E-05	0
7	1	0.210E-05	0.148E-06	0
8	0	0.000E+00	0.156E-07	0

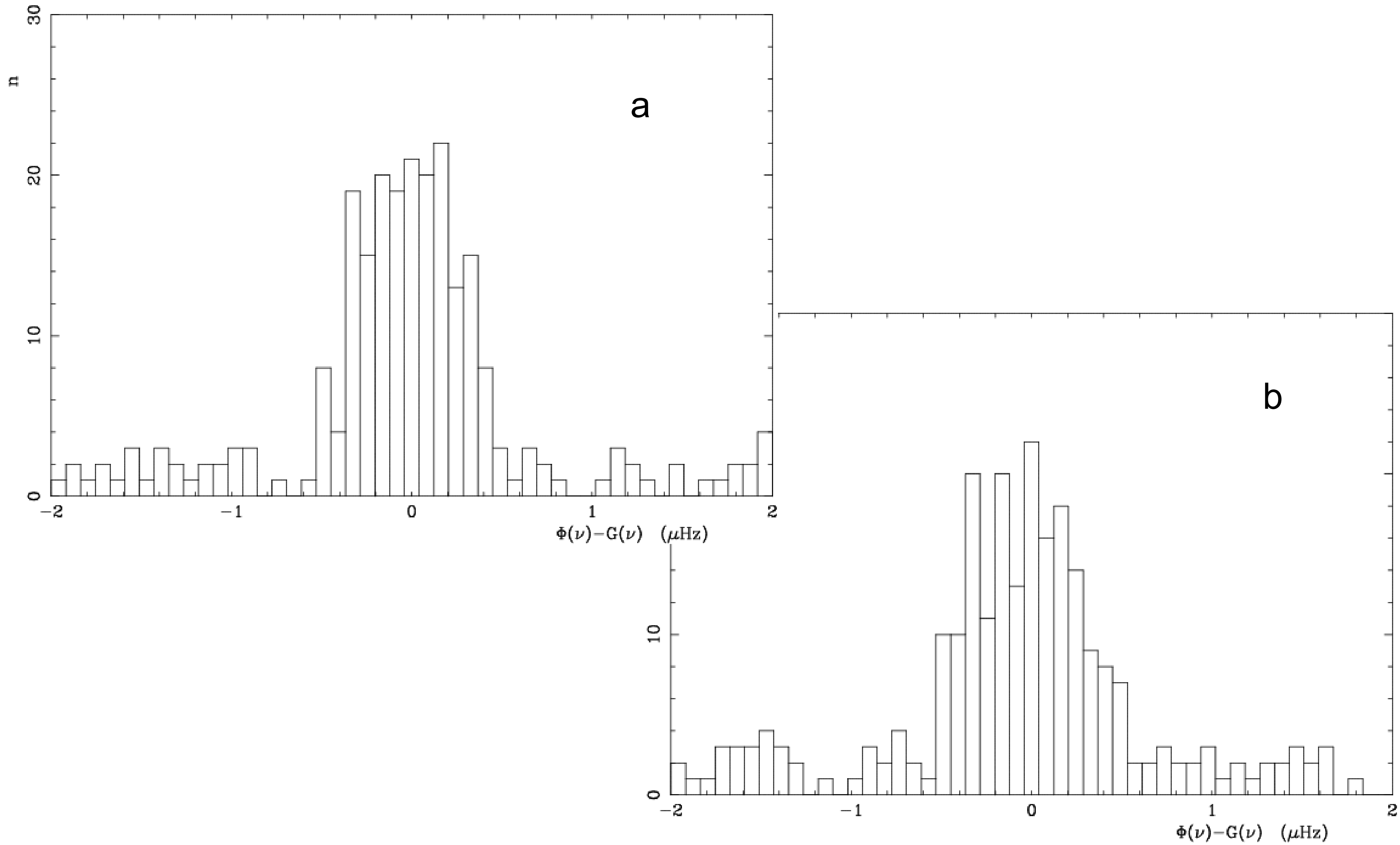
For the higher values of n, the number D(n) of detections is larger than predicted

The experiment is made for a set of values of W, in each we select the frequency for the larger n (W from 3 to 10 months)
We obtain a discrete function Φ (ν) and we compare it to a numerical g-mode table G(ν)

Inf (Φ (ν) - G (ν)) for a numerical g-mode table calculated for a moderately fast rotating core, giving a mean splitting 1 μHz (part 1) (difference with the nearest frequency for degrees *l = 1,2,3*)



a: Histogram of inf (Φ (ν) - G (ν)) for the numerical g-mode table
b: same histogram, but Φ (ν) is replaced with a random number table
-- similar behaviours

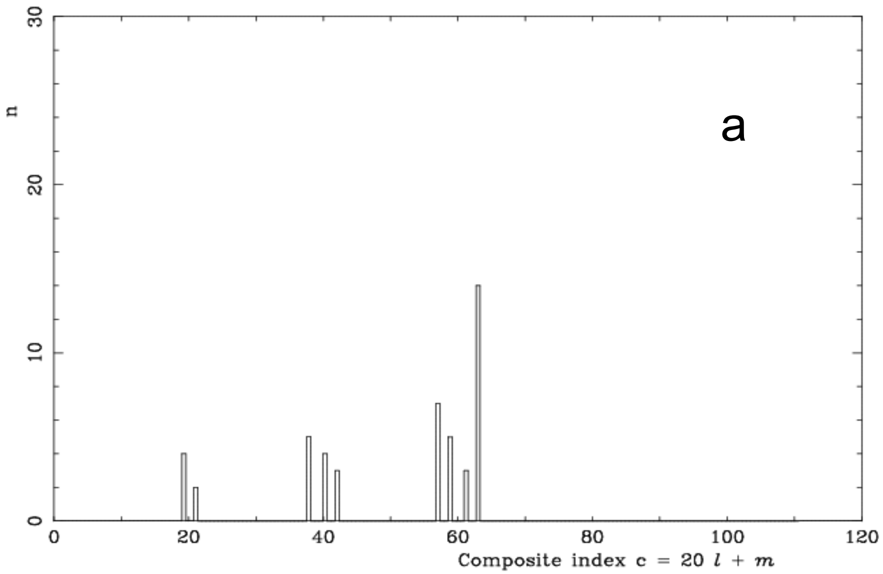


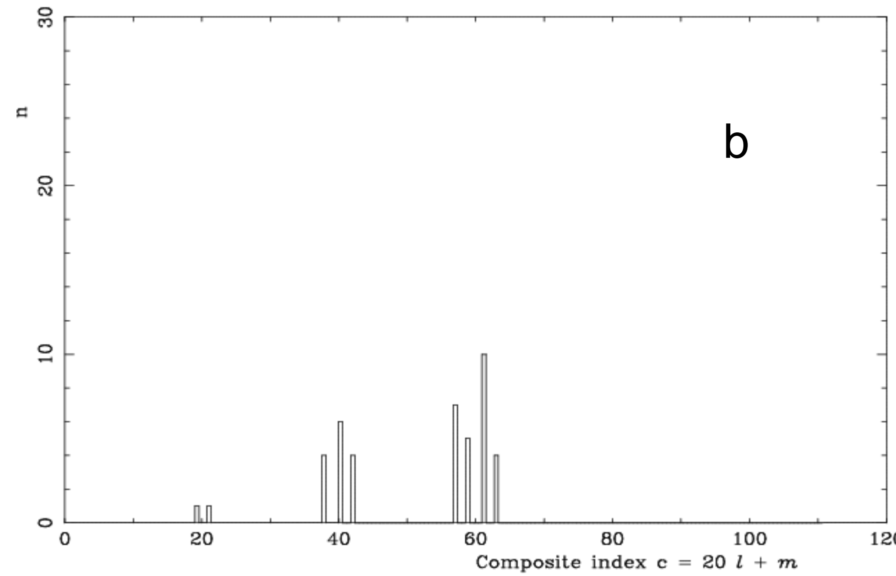
composite histogram of degree *l* and tesseral order *m*

a: modified rotational law (giving the detected 1μHz g-mode splitting)

b: Φ (ν) is replaced with a random number table

c: ... for a large random table



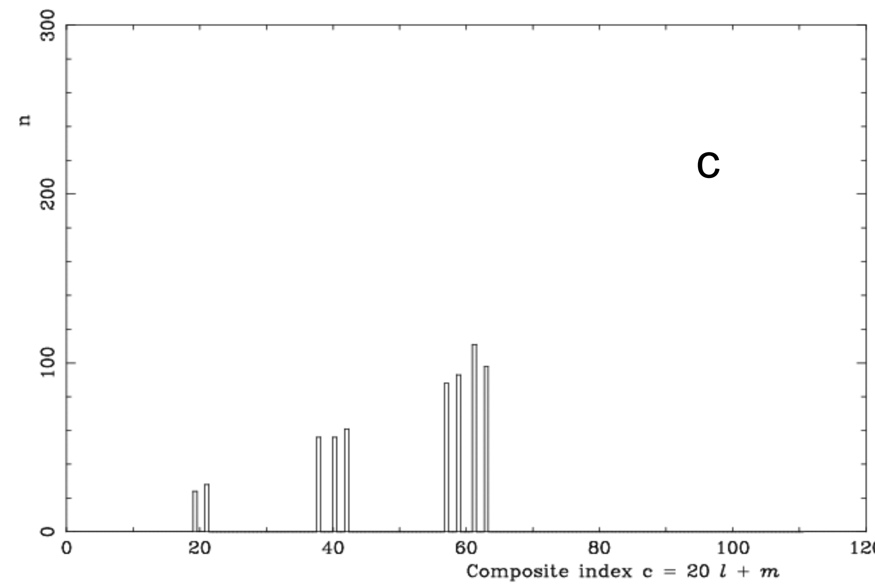


n_i = number of coincidences for degree *l* = i, any *m* and for -0.1 < (Φ (ν) - G (ν)) < 0.1 (μHz)

a: $n_1 / n_2 = 0.5$

b: $n_1 / n_2 = 0.14$

c: $n_1 / n_2 = 0.3$



W	n	*l*	*n*	*m*	frequency μHz (model)	deviation μHz
7.5	6	1	10	-1	62.28430	-0.08983
7.5	4	1	9	-1	68.28530	-0.06629
7.5	4	1	5	1	109.61030	-0.05100
7.5	4	1	2	-1	191.57530	-0.08791
8.0	4	1	14	-1	45.80330	-0.00607
9.0	4	1	5	1	109.61030	0.05780
6.0	6	2	17	-2	63.37260	-0.05161
6.0	6	2	1	2	297.73660	-0.04120
7.0	5	2	21	-2	51.88460	-0.01813
7.5	4	2	24	0	46.64400	0.02047
7.5	4	2	23	2	49.47260	0.00671
7.5	4	2	20	-2	54.36560	-0.01212
7.5	4	2	12	-2	86.93160	-0.05933
7.5	5	2	7	0	135.86400	-0.07269
7.5	4	2	6	0	151.58701	0.02930
7.5	4	2	2	0	256.61700	0.09869
9.0	4	2	6	2	152.46660	0.05721
10.0	4	2	5	-2	169.88060	0.07310

W	n	l	n	m	frequency μHz (model)	deviation μHz
7.5	6	1	10	-1	62.28430	-0.08983
7.5	4	1	9	-1	68.28530	-0.06629
7.5	4	1	5	1	109.61030	-0.05100
7.5	4	1	2	-1	191.57530	-0.08791
8.0	4	1	14	-1	45.80330	-0.00607
9.0	4	1	5	1	109.61030	0.05780

relaxed precision $-0.2 < (\Phi (\nu) - G (\nu)) < 0.2$ (μHz)

6.0	6	1	3	1	153.82831	-0.11981
6.5	6	1	7	1	84.79530	0.17112
6.5	7	1	3	1	153.82831	-0.18991
7.5	4	1	7	-1	84.22030	0.16051
7.5	4	1	7	1	84.79530	0.16962
7.5	6	1	2	-1	191.57530	-0.16991
8.0	4	1	8	-1	75.48330	-0.12217
9.0	4	1	3	1	153.82831	0.16930

W	n	*l*	*n*	*m*	frequency μHz (model)	deviation μHz
7.0	6	3	21	3	74.33490	0.01593
7.5	5	3	31	-3	48.99990	-0.05853
7.5	4	3	27	3	59.23490	-0.08772
7.5	4	3	23	-3	65.31290	-0.03388
7.5	4	3	16	3	94.37490	-0.03714
7.5	4	3	15	-1	97.64830	0.04074
7.5	4	3	13	-1	110.37830	0.06170
7.5	4	3	10	-1	136.59831	0.05429
7.5	4	3	10	3	138.66490	0.02620
7.5	4	3	8	1	162.61830	0.04579
7.5	5	3	4	-3	237.39291	0.07349
8.0	4	3	26	3	61.30490	-0.09093
8.0	4	3	16	3	94.37490	0.00608
8.0	4	3	13	3	112.44490	0.01420
8.0	4	3	4	3	240.16490	-0.09232
8.5	6	3	31	1	51.19830	-0.04857
8.5	4	3	27	1	58.23830	0.00140
8.5	4	3	16	-3	91.17790	0.07979
8.5	4	3	8	3	163.62491	-0.02650
10.0	4	3	9	-1	148.12831	0.07419
6.0	6	3	24	-3	62.72790	-0.01786
6.0	6	3	7	3	179.38490	0.00420
6.0	6	3	1	3	341.99490	0.02701
6.5	5	3	32	3	50.70490	0.09912
6.5	5	3	28	3	57.30490	0.07053
6.5	5	3	7	-3	176.21991	-0.00771
6.5	5	3	5	-1	216.99831	-0.02251
3.0	15	3	5	3	218.91490	-0.01530
4.0	11	3	3	-3	260.27591	0.0058

Results

- up to now, the detection of g modes remains within the random margin.
- a real detection of g modes remain a possibility.
- in that case, the life time is measurable in year or a small number of years.

Perspectives

- several models for rotation
- monitor the time of phase coherences
- monitor the amplitude related to coherent signal

- small number of events: statistic don't help really
- no *l* =1 coincidence for the lower w neither the longer w

Thanks for your attention!