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# Secular Resonances and the Dynamics of Mars-Crossing and Near-Earth Asteroids

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We study by numerical integration the secular dynamical behavior of a sample of 32 Mars-crossing and near-Earth asteroids. We divide the planet-crossing asteroid population into two groups, with semimajor axis $a$ >2 AU and <2 AU. In the former region ($a$>2 AU), a computer-assisted analytical theory allows us to select 24 objects which are inside or close to the main secular resonances. The numerical computation of their evolution shows that the global dynamical picture is characterized by the existence of "fast tracks" (strong and rapid changes in the eccentricity due to resonances) and "slow tracks" (random walk in semimajor axis due to close approaches to the inner planets). The $\nu_6$ secular resonance appears to be a common and very effective fast-track mechanism. A solar collision is the most probable end-state of bodies on the fast-track orbits, but hyperbolic ejections following Jupiter encounters are also observed. In the inner region ($a$<2 AU), no analytical theory can be applied and we integrate eight orbits of NEAs which had shown secular trends in eccentricity and/or inclination in previous (shorter) integrations. Here also evolutions mainly affected by resonances and by close encounters can be distinguished, although the interplay of the two kinds of effects appears to be complex. The existence of secular resonances in this region is pointed out for the first time, but further investigations are needed to understand their locations and strengths. © 1995 Academic Press, Inc.

## 1. INTRODUCTION

The dynamical evolution of Mars-crossing (MCs) and near-Earth asteroids (NEAs) probably represents the most difficult problem in celestial mechanics. These bodies have lifetimes which last, at most, several percent of the age of the Solar System, and this implies that they are a quasi-stationary population supplied by one or more external sources. Actually, a very plausible possibility is that most of these bodies are fragments of main-belt asteroids injected into a mean motion or secular resonance, which has pumped up the orbital eccentricity, forcing the perihelion distance to become smaller than the aphelion distances of the inner planets (Williams 1973, Wisdom 1985, Scholl and Froeschlé 1991, Wetherill 1985, 1987; Farinella *et al.* 1993, Morbidelli *et al.* 1994). According to this scenario, at their very birth the NEA and MC orbits are resonant; moreover, as soon as their eccentricities become large enough that the orbits cross those of the inner planets, the orbital parameters are modified in a random-walk fashion by a sequence of planetary close encounters. Thus, we have a complex interplay between close encounters and resonances, which can yield a variety of unexpected outcomes, including cometary-type orbits (Valsecchi *et al.* 1995) and even solar collisions (Farinella *et al.* 1994). For a detailed analysis of a specific example [the NEA (4179) Toutatis], we refer to Michel *et al.* (1995).

The effect of secular resonances on the dynamical evolution of NEAs and MCs so far has been little known. There are two main reasons for this: (i) most analytical studies on secular resonances (e.g., Williams and Faulk-



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ner 1981, Morbidelli and Henrard 1991a,b, Knežević *et al.* 1991) have been focused on the main asteroid belt, where these resonances are well known to play an important role, and have not addressed orbits with high eccentricities and/or semimajor axes <2 AU; (ii) numerical studies on the dynamics of NEAs (Milani *et al.* 1989) have been performed over time spans of at most a few times $10^5$ years, too short for clearly detecting and assessing the slow, long-periodic changes of the elements associated with secular perturbations. As a consequence, the Monte Carlo statistical models developed to study NEA origin and evolution (Wetherill 1979, 1988) have used very simple and somewhat artificial models for the effects of secular resonances. The validity of the corresponding results is questionable because, as we shall show, these resonances turn out to affect in a very strong way the orbits of a significant fraction of the known NEAs and MCs. Moreover, a number of important open questions have still to be addressed in modeling the dynamical evolution of NEAs and MCs. For instance: how wide are the "resonant strips" in orbital element space? Which values of the eccentricity and/or inclination can be reached while inside the resonances and on which time scale? Is there any coupling/overlapping between different resonant effects? How effective are planetary encounters in moving the orbits in and out of the resonant zones?

Most of these questions are now mature enough for quantitative study. Recent theoretical work (Morbidelli 1993, Morbidelli *et al.* 1994) has resulted in a much better understanding, and also a satisfactory semiquantitative modeling, of the effects of secular resonances. However, as soon as the orbits become planet-crossing, the analytical models lose their validity, and in this case neither the secular effects of the inner planets nor the encounters with them can be taken into account by such theories. On the other hand, current computer technology allows us to easily integrate numerically the orbits of NEAs and MCs within a realistic dynamical model over several millions of years. Although NEA/MC orbits are in most cases strongly chaotic (with Lyapunov times $\approx 10^2$ years) and therefore cannot be predicted deterministically over millions of years, numerical integrations can provide a useful sampling of different orbital behaviors and evolutionary patterns and also yield quantitatively meaningful statistical results on the evolution of an entire population of such small bodies (see, e.g., the recent work on the short-period comets by Levison and Duncan 1994).

Thus, we believe that new numerical studies supplemented by the insights provided by analytical theories have the potential of leading to improved models for the origin and evolution of NEAs and MCs and of course also to a better understanding of the dynamical history of meteorites, which can be seen as small NEAs having hit Earth. Therefore, we have decided to undertake such an extensive study of the dynamics of NEAs and MCs and to focus in particular on objects which *a priori* appear likely to be strongly affected by secular resonances.

The results of this study demonstrate clearly that secular resonances are an essential ingredient in the dynamics of a significant number of NEAs and MCs and that neglecting them or modeling them in an oversimplified way can easily lead to wrong conclusions on the speed and the effectiveness of the different dynamical pathways between the main asteroid belt and the Earth-crossing orbits. We will also show from a dynamical point of view, that the population of NEAs and MCs is extremely diverse, with many mechanisms at work for different objects—and often for the same object at different times—and with many evolutionary patterns; these range from quasi-stable orbits weakly affected by random encounters, with likely lifetimes >10 Myr, to very rapidly evolving orbits, which lead to a collision with the Sun or to a hyperbolic ejection within 1 Myr. Taking into account this diversity will clearly be essential in improving our understanding of the "demography" of the NEA and MC population and of the interplays between dynamical and physical evolution processes (e.g., due to collisions with main-belt asteroids and/or other NEAs).

The remainder of this paper is organized as follows. In Section 2, we will describe the procedure by which we have chosen our samples of near-resonant NEAs and MCs, and we will provide some details on the algorithms used to perform the integrations. In Section 3, we will discuss the results, providing a general picture on NEA and MC dynamics as we have come to understand it and showing that NEA and MC orbits can be classified into several classes having different long-term behaviors as a consequence of a variety of mechanisms. In Section 4, we shall discuss some implications of our work and some outstanding remaining open problems, and we will try to extract from all our numerical integrations a unitary (if still sketchy) picture of the dynamics of NEAs and MCs.

## 2. THE CHOICE OF THE SAMPLE OF OBJECTS FOR THE NUMERICAL INTEGRATIONS

The dynamics of planet-crossing objects presents very difficult problems from the theoretical point of view. The possibility of having a collision with a planet, in particular, introduces a singularity in the equations of motion, such that the usual theories, which describe the secular evolution of orbits by means of averaged equations, cannot be applied. As a matter of fact, a close approach with a planet produces strong and fast changes of the osculating elements of a small body; in a long-term numerical integration of any such orbit, these changes appear as "chaotic jumps," which affect in the most apparent way the semimajor axis. These jumps can be defined as chaotic since they are very sensitive to the preencounter orbital parameters, due to the fact that the dynamics is hyperbolic with respect to a planetocentric reference system. Thus analytical theories can provide only a limited contribution to the the investigation of the dynamical evolution of asteroids having either a small semimajor axis (roughly speaking, $a<2$ AU), or an eccentricity large enough (say, $e>0.3$) to cross a planetary orbit even though the semimajor axis has a typical main-belt value. On the other hand, in contradiction with common beliefs, a large eccentricity alone (i.e., in absence of planetary encounters) would not be an unsurmountable problem, at least for computer-assisted analytical theories which do not use classical expansions in powers of $e$ of the equations of motion. This means that, while keeping in mind the above-mentioned problems, and without aiming at being very accurate from the quantitative point of view, some analytical work can still be of some help and, indeed, has been the starting point for the realization of the numerical work described later in this paper.

For what concerns the objects with typical main-belt semimajor axes (2 AU $< a <$3.5 AU) and large eccentricities, as a first approximation we have simply neglected the effect of the inner planets—or, more exactly, we have assumed that this effect is negligible during most of the time, except during the close planetary approaches, which produce just a sort of random walk in $a$ (slow if the orbit approaches only Mars, faster and with larger "steps" if

**TABLE I**
*Osculating Elements for the Two Sets of Orbits Integrated*

| | Object | $a$ (AU) | $e$ | $i$ (deg) | $\Omega$ (deg) | $\omega$ (deg) | $M$ (deg) | $q$ (AU) | $Q$ (AU) | $g$ ("/yr) | $s$ ("/yr) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 512 | Taurinensis | 2.1904112 | 0.2535641 | 8.75060 | 107.32440 | 248.72538 | 173.62722 | 1.635 | 2.746 | 30.23105 | -38.58761 |
| 1468 | Zomba | 2.1966066 | 0.2702309 | 9.94318 | 309.16824 | 22.66893 | 249.63730 | 1.603 | 2.790 | 28.31319 | -39.43646 |
| 2368 | Beltrovata | 2.1046371 | 0.4139238 | 5.25002 | 287.83485 | 42.22512 | 15.35148 | 1.233 | 2.976 | 28.25427 | -46.88543 |
| 3102 | 1981 QA | 2.1527552 | 0.4480294 | 8.41034 | 172.41740 | 154.28422 | 219.48778 | 1.188 | 3.117 | 28.79969 | -53.25403 |
| 3402 | Wisdom | 2.1323780 | 0.2788066 | 4.85231 | 358.25166 | 302.98332 | 252.59714 | 1.538 | 2.727 | 29.28172 | -38.15851 |
| 3551 | Verenia | 2.0917727 | 0.4879817 | 9.51636 | 174.04360 | 193.00373 | 26.79494 | 1.071 | 3.113 | 29.60662 | -55.10255 |
| 3671 | Dionysius | 2.1944675 | 0.5430472 | 13.62545 | 82.45823 | 203.67180 | 222.82348 | 1.003 | 3.386 | 27.94984 | -68.21976 |
| 3833 | 1971 SC | 2.1972065 | 0.3884573 | 11.99404 | 170.39450 | 173.68387 | 204.31528 | 1.344 | 3.051 | 28.89682 | -49.72328 |
| 3858 | Dorchester | 2.1894700 | 0.2430236 | 7.71386 | 348.20507 | 335.01767 | 4.27808 | 1.657 | 2.722 | 29.82207 | -38.13554 |
| 4451 | Grieve | 2.6024787 | 0.3864053 | 27.70687 | 219.93517 | 109.47999 | 349.59787 | 1.597 | 3.608 | 5.00627 | -43.89558 |
| 4596 | 1981 QB | 2.2404816 | 0.5179151 | 37.12958 | 154.57624 | 248.24242 | 118.73248 | 1.080 | 3.401 | -3.99524 | -29.69390 |
| 4924 | Hiltner | 2.1533769 | 0.2150000 | 1.72296 | 180.31817 | 148.20617 | 130.25905 | 1.690 | 2.616 | 30.80017 | -35.87689 |
| 5620 | 1990 OA | 2.1590878 | 0.4221525 | 7.83519 | 129.00431 | 152.96385 | 288.81366 | 1.248 | 3.071 | 29.70283 | -51.36609 |
| 5646 | 1990 TR | 2.1426275 | 0.4373981 | 7.91438 | 14.30206 | 335.39296 | 270.76885 | 1.205 | 3.080 | 28.79403 | -52.01580 |
| 5731 | 1988 VP4 | 2.2626054 | 0.6531056 | 11.65813 | 282.82060 | 215.56762 | 51.83858 | 0.785 | 3.740 | 37.91164 | -106.51275 |
| 5870 | 1989 CC1 | 2.7917118 | 0.4205102 | 28.92055 | 154.67781 | 271.04718 | 345.66849 | 1.618 | 3.966 | 1.61596 | -48.23022 |
| | 1972 RB | 2.1489126 | 0.4865138 | 5.21922 | 177.52718 | 152.34418 | 169.86945 | 1.103 | 3.194 | 29.54621 | -58.22787 |
| | 1988 NE | 2.1812727 | 0.4421886 | 9.93090 | 254.27136 | 354.78182 | 157.68025 | 1.217 | 3.146 | 28.60253 | -54.67605 |
| | 1988 PA | 2.1557048 | 0.4093681 | 8.22999 | 162.41925 | 137.03659 | 149.26161 | 1.273 | 3.038 | 28.87423 | -49.00748 |
| | 1989 DA | 2.1616618 | 0.5437576 | 6.44978 | 349.77929 | 138.75399 | 86.03787 | 0.986 | 3.337 | 29.04843 | -67.01475 |
| | 1991 VC | 2.1623898 | 0.2898571 | 7.32990 | 54.07396 | 252.20344 | 197.68439 | 1.536 | 2.789 | 29.60499 | -39.71721 |
| | 1992 RD | 2.1611215 | 0.1199447 | 26.93938 | 130.51744 | 140.07419 | 115.87943 | 1.902 | 2.420 | 11.50992 | -26.29583 |
| | 1992 SY | 2.2093046 | 0.5501821 | 8.02997 | 6.32449 | 115.02410 | 341.39336 | 0.994 | 3.425 | 29.35959 | -70.40849 |
| | 1992 SZ | 2.1769247 | 0.4598675 | 9.27945 | 4.43656 | 314.67113 | 47.34767 | 1.176 | 3.178 | 28.54111 | -55.56233 |
| 1620 | Geographos | 1.2447902 | 0.3354166 | 13.32576 | 337.39084 | 276.62047 | 125.93282 | 0.827 | 1.662 | | |
| 1862 | Apollo | 1.4711036 | 0.5599483 | 6.35634 | 35.93390 | 285.64358 | 72.89434 | 0.647 | 2.295 | | |
| 1864 | Daedalus | 1.4608908 | 0.6148305 | 22.18099 | 6.79272 | 325.45154 | 151.54717 | 0.563 | 2.359 | | |
| 2101 | Adonis | 1.8754319 | 0.7643423 | 1.35909 | 351.12130 | 41.84797 | 172.43832 | 0.442 | 3.309 | | |
| 2135 | Aristaeus | 1.5998117 | 0.5031602 | 23.03802 | 191.45112 | 290.65151 | 48.78470 | 0.795 | 2.405 | | |
| 2201 | Oljato | 2.1754056 | 0.7109842 | 2.51585 | 76.91087 | 95.93654 | 70.10462 | 0.629 | 3.722 | | |
| 3757 | 1982 XB | 1.8352899 | 0.4462655 | 3.87539 | 75.11014 | 16.90412 | 93.16202 | 1.016 | 2.654 | | |
| 3908 | 1980 PA | 1.9241653 | 0.4589216 | 2.16990 | 262.06028 | 125.55014 | 284.94091 | 1.041 | 2.807 | | |
| 6063 | 1984 KB | 2.2164589 | 0.7643633 | 4.84604 | 169.95677 | 336.50748 | 294.71365 | 0.522 | 3.911 | | |
| | 1977 VA | 1.8641821 | 0.3941888 | 2.97322 | 224.75605 | 172.11541 | 67.71428 | 1.129 | 2.599 | | |

The first group refer to epoch JD 2449000.5, the second to JD 2449200.5. The last two columns contain the frequencies computed by computer assisted analytic methods (see section 2).

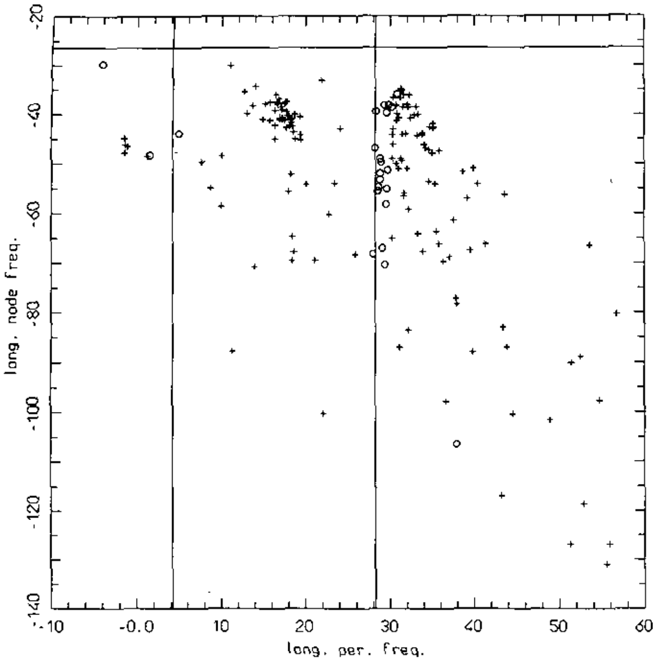


FIG. 1. The result of the computation of proper elements for the main belt Mars crossers in the frequency space (g,s). Crosses denote asteroids for which the computation of proper elements succeeds; circles denote asteroids for which the algorithm diverges. Solid lines denote the three main secular resonances $g=g_5$, $g=g_6$ and $s=s_6$.

it approaches Earth and/or Venus) superimposed to the secular dynamics induced by the giant planets. In order to investigate these secular dynamics, we have used the program for the derivation of asteroid proper elements developed by Lemaître and Morbidelli (1994), which works properly for any value of the eccentricity (actually, up to $e \approx 0.7$, for technical reasons). If for a given orbit the program provides proper elements, this means that, in absence of the inner planets, the orbit would be a regular one; if on the contrary the algorithm for the computation of proper elements does not converge, one can conclude that the orbit is affected by some resonance (the program points out which one) and one can expect that resonant phenomena occur.

Two classes of resonances are important for the dynamics of asteroids: mean motion resonances and secular resonances. The former ones are the commensurabilities between the orbital period of revolution around the Sun of the asteroid and that of Jupiter; the latter ones are the resonances between the precession rates of the asteroid's orbit and the precession rates of the orbits of the giant planets (mainly Jupiter and Saturn). More precisely, the Sun–Jupiter–Saturn system is characterized by three main secular frequencies: $g_5$ (the average frequency of precession of Jupiter's perihelion), $g_6$ (the average frequency of precession of Saturn's perihelion), and $s_6$ (the average frequency of precession of Jupiter's and Saturn's nodes). The main secular resonances occur when the frequency $g$ of the longitude of perihelion of an asteroid is equal to $g_5$ or $g_6$ or when the frequency $s$ of the node of an asteroid is equal to $s_6$. In Williams' (1969) notations, these resonances are called $\nu_5$, $\nu_6$, and $\nu_{16}$, respectively. Another secular resonance, present at large inclinations, is the so called *Kozai resonance* (Kozai 1962), which occurs when the precession of the argument of perihelion of the asteroid stops, namely when $g = s$.

Recently, Morbidelli (1993) has shown that among the secular resonances, the $\nu_6$ one is particularly strong, since it can rapidly pump up the eccentricity from zero to very large values, 0.6 and even more; in some cases the eccentricity can become so large (about 0.995) that the body falls into the Sun (Farinella *et al.* 1994). Moreover, Morbidelli *et al.* (1994) pointed out that some asteroids are very close in proper element space to the $\nu_6$ resonance and as a consequence can easily inject into this resonance many collisional fragments; they concluded that these asteroids are likely to be efficient sources of meteorites and the $\nu_6$ resonance one of the main mechanisms of transport of material from the asteroid belt to Earth. On the other hand, Morbidelli (1993) has shown that $\nu_5$ and $\nu_{16}$ resonances are not so effective in changing the eccentricity to provide an important transport mechanism to Mars and Earth (Table I; see also the numerical results of Scholl and Froeschlé 1986, 1990).

As for the mean motion resonances, objects in the 3/1, 5/2 and 4/1 resonances with Jupiter—which correspond to well-known Kirkwood gaps in the distribution of main-belt asteroids—have been shown by many authors (Wisdom 1983, Yoshikawa 1989, 1991, Scholl and Froeschlé 1991, Hahn *et al.* 1991) to frequently reach eccentricities high enough to cross the orbit of Mars. More recently, Moons and Morbidelli (1995) have pointed out that the overlapping of secular resonances inside these mean motion commensurabilities is at the origin of large-scale chaos, and that as a consequence the eccentricity can be pumped up to cross even Earth's orbit in an efficient manner.

Figure 1 summarizes the results of our computations of proper elements for the main-belt Mars-crossers, obtained by neglecting the existence of the inner planets. In more detail, this is the procedure that we have followed. We took a list of Mars-crossing objects updated to December 31, 1992, which consists of 430 objects taken from the *Minor Planet Circulars*. Then, we integrated all the orbits back to the common Julian date 2449000.5 using the RA15 integrator (Everhart 1985) and taking into account the perturbations of all planets except Pluto. Then, we selected all objects with $a$ between 2 and 3 AU. Since the computation of the mean elements is not accurate at large eccentricities (Milani and Knežević 1990), we simply computed the elements of these bodies with respect to the invariant plane of the Solar System and assumed these new values as "mean elements." (This may appear as a very crude approximation, but we recall that our aim here is just to distinguish between those objects which are close to the secular resonances and those which are not.) Then we computed the proper elements of the selected bodies using the program developed by Lemaître and Morbidelli (1994). This step introduces some new selection criteria. First of all, for technical reasons related to working of the proper element program, one has to discard the orbits with $e > 0.7$ and 2.679 AU $< a <$ 2.731 AU. The former condition eliminates 11 objects, 7 of which have been integrated numerically by Valsecchi *et al.* 1995 [including (2201) Oljato and (6063) 1984 KB, which we have integrated again here anyway—see Table I for reference]. The second condition eliminates 6 objects, which are near the 8/3 mean motion resonance with Jupiter. Moreover, an additional selection is imposed by the fact that the proper element program cannot compute the frequencies of bodies which are too close to a strong mean motion resonance, mainly the 3/1 and 5/2 ones. We have found 3 NEAs very close to the 5/2 and 15 very close to the 3/1 resonance, and these asteroids have been also discarded. At the end of this selection procedure, there remain 182 asteroid orbits to which the proper element algorithm could be applied.

In Fig. 1 we have plotted, rather than the commonly used proper semimajor axes, eccentricities, and inclinations, the proper frequencies $g$ and $s$, in order to point out in a clearer way the role of secular resonances. The crosses denote the proper frequencies of the objects for which proper elements can be computed. These objects would have a regular orbit if the inner planets did not exist. The circles, on the other hand, correspond to the approximate frequencies of those bodies for which proper elements cannot be computed, since they are too close to a resonance. The two vertical lines show the positions of the $\nu_5$ secular resonance at $g = 4.257$ arcsec/year and the $\nu_6$ secular resonance at $g = 28.245$ arcsec/year; the horizontal line denotes the $\nu_{16}$ resonance at $s = -26.345$ arcsec/year. It is apparent that most of the circles lie along these lines: in particular, 21 are very close to the $\nu_6$ resonance, 2 are close to the $\nu_5$, and 1 is close to the $\nu_{16}$ resonance. This finding confirms our previous studies that the $\nu_6$ resonance plays a major role for the origin of NEAs and MCs, and that the other two resonances are much less important. In effect in our previous work (Morbidelli 1993, Morbidelli *et al.* 1994) studying the dynamics of fictituous objects in the $\nu_6$ resonance, we concluded that this resonance can be a major transport channel for main belt asteroid fragments to planet-crossing orbits. The fact that we can now identify several real MCs and NEAs close/inside the $\nu_6$ resonance confirms the real importance of this transport route. There is one circle which is far from the lines corresponding to the main secular resonances; it corresponds to the asteroid (5731) 1988 $VP_4$, and the nonconvergence of the proper elements algorithm in this case is due to the interplay between secular forcing terms and the nearby presence of the 3/1 mean motion resonance, a mechanism likely to cause chaotic behavior. As we shall see in Section 3, our numerical integration of this orbit confirms this prediction.

If we now take into account the effect of the inner planets, we expect that the random walk of the semimajor axis due to close approaches can shift the asteroids in the frequency plane of Fig. 1, and, as a consequence, objects can be sporadically injected into a resonance or extracted from it. This interplay between dynamics induced by the giant planets and semimajor axis random walk induced by the inner planets cannot be described by analytical models for the reasons discussed earlier. This is why, in the following, we have integrated all the 23 orbits corresponding to circles in Fig. 1. The behavior of these orbits is discussed in Section 3.1. All of them can be qualitatively understood by keeping in mind that the semimajor axis random walk can displace a planet-crossing object with respect to the resonance that it lies close to.

For what concerns the bodies lying inside the inner edge of the main belt, namely with $a<2$ AU, the game of neglecting as a first approximation the inner planets cannot work even qualitatively. Indeed, the direct effect of the inner planets is now important all along the asteroid orbit. As an example, if one neglects the inner planets, no relevant secular resonances occur for $a < 2$ AU. On the other hand, in our numerical integrations we have found some cases which are temporarily in the $\nu_5$ and $\nu_{16}$ resonances. Therefore, the investigation of the dynamics of these objects has to be done, at least for the present time, by pure numerical integrations. Among the 191 Mars-crossers with $a < 2$ AU, we have selected 8 objects which had showed clear secular trends in the outputs of the 200,000 year integrations of Milani *et al.* (1989). The results are described in Section 3.2. We plan to integrate more NEAs and MCs with $a<2$ AU in the next stage of our work.

Finally, we are going to add a short discussion on the case of asteroid 1992 RD, since this object, although it is not currently an Earth- or Mars-crosser, is locked in the $\nu_{16}$ resonance, making it a good candidate to become a MC in a relatively close future.

As a summary, we have listed in Table I all the asteroids that we have integrated numerically, with the corresponding initial conditions and the theoretically computed secular frequencies, in the cases when such a computation is possible. All the numerical integrations discussed in this paper have been carried out with either a Bulirsch–Stoer variable-stepsize technique (Stoer and Bulirsch 1980) or

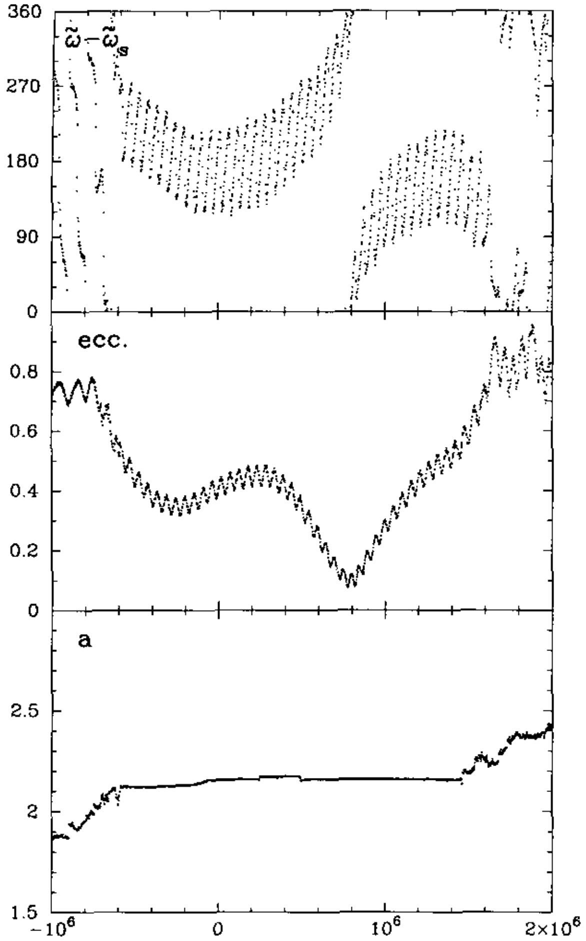


FIG. 2. The evolution of 1988 PA. We remark the large secular changes of the eccentricity due to the libration of $\tilde{\omega}$-$\tilde{\omega}_S$ about 0° in the $\nu_6$ resonance.

a 15th-order Radau integrator (Everhart 1985); both programs have been optimized for dealing accurately with planetary close approaches, and the use of both integration techniques allowed us to carry out some interesting comparisons between the corresponding results (see Section 3). All planets except Pluto and Mercury (due to their small masses) were taken into account in the dynamical model. We also made a few integrations taking into account Mercury, to check that the corresponding effects are not important. The integration time span extended at least from $t = -1$ Myr (in the past) to $t = 1$ Myr (in the future). In some cases we extended the integration by 1 or 1.5 Myr farther in the past and/or the future, to find out the outcome of strong secular trends in the orbital elements.

As in every such numerical work, it has to be stressed that owing to the strongly chaotic nature of the orbits, the results of the numerical integrations cannot be seen as deterministic predictions of the evolution of real objects over a long span of time: the reason is that planet-crossing orbits are so strongly chaotic (i.e., sensitive to the chosen initial conditions, physical model of the Solar System, integration algorithm, computer round-off features, etc.) that after a time span of the order of $10^3$ years, much shorter than our integration times, they "lose the memory" of the corresponding real object. If one integrates the "same object" but with slightly different starting conditions (within the observational errors), or including one more planet (e.g., Mercury), or using a different integration code or the same code but on another computer, one gets orbital evolutions which may be very different from each other on times much longer than $10^3$ years (provided that the orbit is planet-crossing). So, the reader may wonder, what is the meaning of the integrations? The reply is that they provide valuable qualitative and/or statistical information on the most common patterns of orbital evolution and behavior, as there are good reasons to think

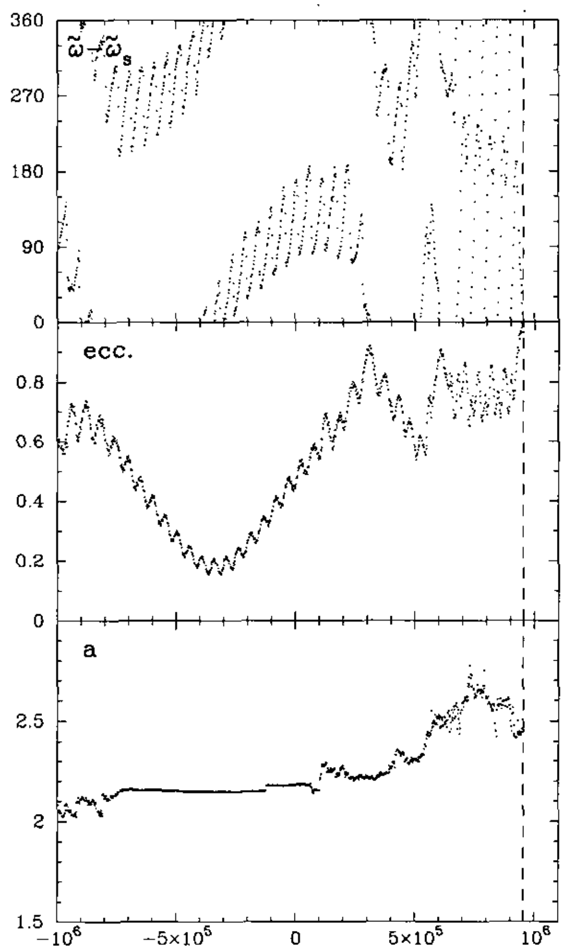


FIG. 3. The evolution of 1988 NE. The libration of $\tilde{\omega}$-$\tilde{\omega}_S$ about 0° in the $\nu_6$ resonance pumps down and up again the eccentricity; the changes in the semimajor axis take the asteroid away from the $\nu_6$ resonance. A final jump in the eccentricity takes the asteroid to collide with the Sun.

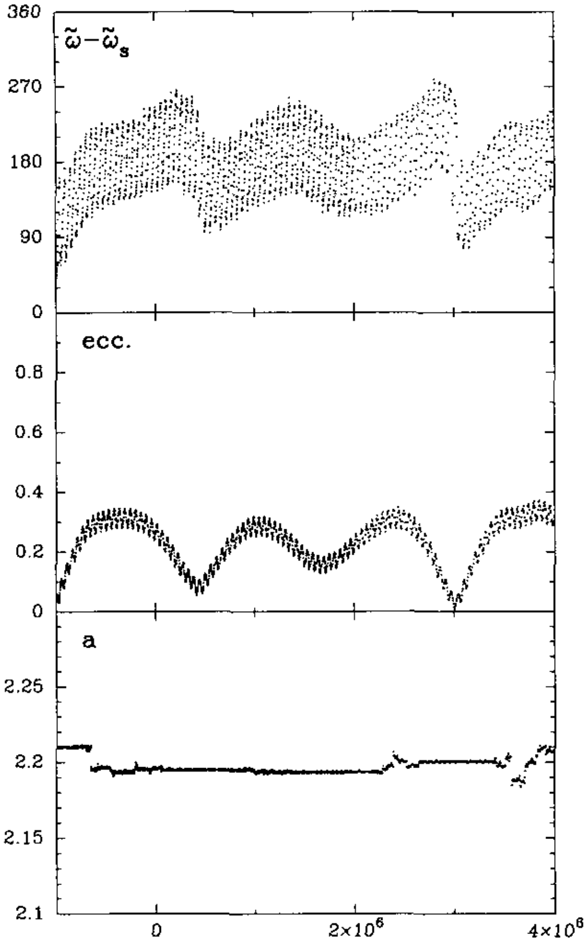


FIG. 4. The evolution of 1468 Zomba. The critical angle of the $\nu_6$ resonance is blocked around 180° and the eccentricity does not change much. (see text for comments).

that the qualitative features (such as resonance lockings, frequency of close planetary encounters, etc.) will be the same in a large enough sample of real and integrated orbits. This applies for instance to the Farinella *et al.* (1994) finding on the relatively high frequency of solar collisions. For the same reason, the integrations backward in time should not be seen as the real histories of the real bodies, but just as a simple way of doubling the size of the sample and improving the statistics; for a detailed discussion on this point, we refer to the recent paper by Levison and Duncan (1994) on the evolution of short-period comet orbits.

## 3. RESULTS OF THE NUMERICAL INTEGRATIONS: A GENERAL PICTURE

In this section we are going to analyze and discuss the different dynamical evolution patterns obtained in the numerical integrations. First we consider the bodies with a semimajor axis >2 AU, for which the proper element algorithm does not converge because they are located in or near a resonance (the circles in Fig. 1). Then we will separately discuss the objects with $a$ <2 AU.

### *3.1 Near-Resonant Objects*

Depending on the dominant dynamical mechanism at work, we can divide this sample into four subgroups.

*3.1.1. Secular resonance $\nu_6$.* Among the 21 asteroids which have their perihelion frequency $g$ nearly equal to $g_6$, we find four kinds of behavior.

(i) The evolution of asteroids (2201) Oljato, 1992 SZ, (5646) 1990 TR, (3833) 1971 SC, 1988 PA, 1988 NE, 1972 RB, (3551) Verenia, (6063) 1984 KB, (5620) 1990 OA is strongly influenced by the $\nu_6$ secular resonance. They are locked always or—in some cases—temporarily in the resonance, as shown by the fact that the critical argument

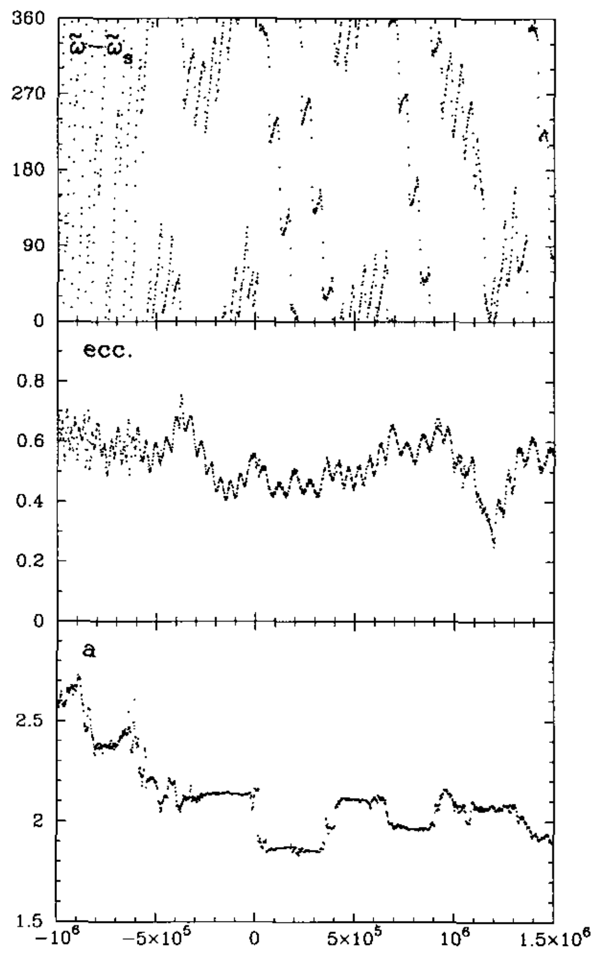


FIG. 5. The evolution of 1989 DA. Several close encounters with Earth occur, which force large changes of a. As a consequence the asteroid is displaced with respect to the $\nu_6$ resonance and the critical angle $\tilde{\omega}$-$\tilde{\omega}_S$ alternates between circulations and librations.

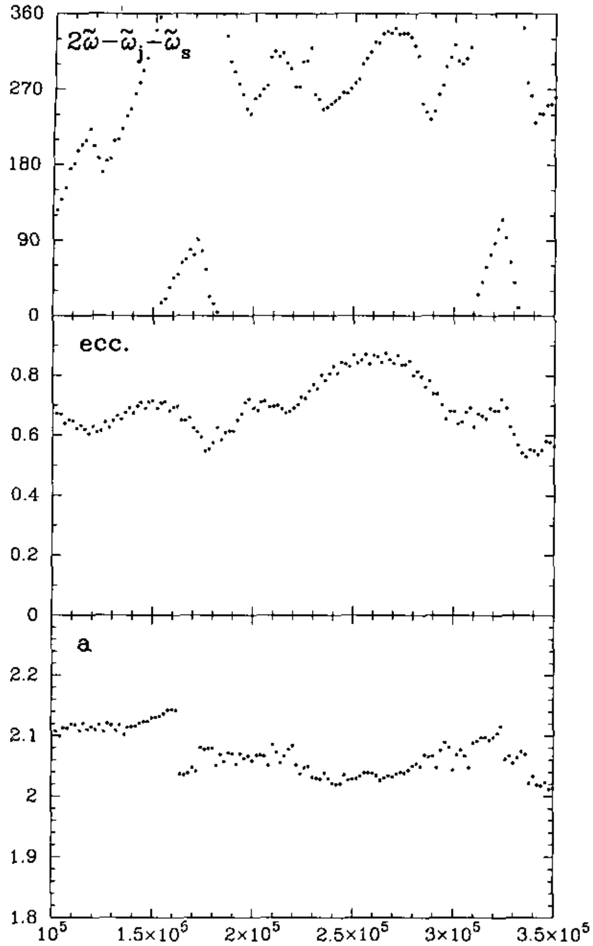


FIG. 6. The evolution of the asteroid (3671) Dionysius between $10^5$ and $3.5\times 10^5$ years. Due to the secondary resonance $2g-g_5-g_6=0$ (the critical angle of which $2\tilde{\omega}-\tilde{\omega}_J-\tilde{\omega}_S$ librates) a burst in the eccentricity occurs up to e=0.82.

$\tilde{\omega}-\tilde{\omega}_S$ ($\tilde{\omega}_S$ being Saturn's longitude of perihelion) librates around 0°. Consequently, their eccentricity can be increased drastically (up to ≈0.8), so that they become Earth-crossers and even Venus-crossers (see Fig. 2, showing the behavior of the orbit of 1988 PA). The eccentricity changes are so drastic that seven of these asteroids fall into the Sun within the integration time span, their perihelion distance reaching 1 $R_\odot$. This happens either in the backward integrations [as in the cases of (3551) Verenia and (5620) 1990 OA, which have lifetimes shorter than 1 Myr] or in the forward integrations [as in the cases of (2201) Oljato, 1992 SZ, (3833) 1971 SC, (6063) 1984 KB and 1988 NE, the first three having lifetimes between 1 and 2 Myr, the last two shorter than 1 Myr].

As described in Farinella *et al.* (1994), two dynamical mechanisms involving the $\nu_6$ secular resonance can lead NEAs to hit the Sun. The first mechanism [active for (6063) 1984 KB, 1992 SZ, (5620) 1990 OA] requires the $\nu_6$ resonance alone: the resonance directly pumps up the eccentricity to almost unity, as illustrated by Fig. 1 in Farinella *et al.* and this results into a solar collision. The second mechanism [(2201) Oljato, (3833) 1971 SC, 1988 NE, (3551) Verenia] requires also other secular forcing terms besides the $\nu_6$ resonance: the asteroids stay only temporarily in the resonance, with the eccentricity reaching up to 0.8, and then leave the resonance due to planetary close encounters; subsequently, secular oscillations of $e$ with arguments $\tilde{\omega}-\tilde{\omega}_J$, $\tilde{\omega}-\tilde{\omega}_S$ and $2\tilde{\omega}-\tilde{\omega}_S-\tilde{\omega}_J$ can further decrease the perihelion distance until a solar collision occurs (see Fig. 3, referring to 1988 NE). We also recall that a collision with the Sun is not the only possible fate of these $\nu_6$-resonant objects: the large eccentricity can also lead them to a hyperbolic ejection following a close encounter with Jupiter. For instance, this is the case of (3833) 1971 SC, which, in our backward integration, is ejected from the Solar System at $t = -0.726$ Myr.

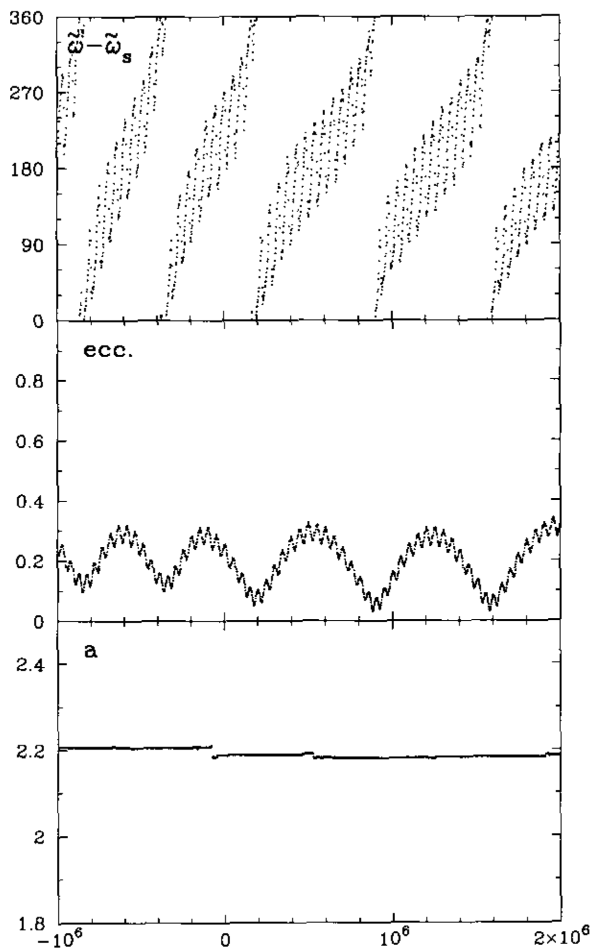


FIG. 7. The evolution of the asteroid (512) Taurinensis. The asteroid is close but outside of the $\nu_6$ resonance ($\tilde{\omega}-\tilde{\omega}_S$ circulates slowly). As a consequence the eccentricity exhibits moderate regular oscillations which take the asteroid to be temporary Mars-crosser.

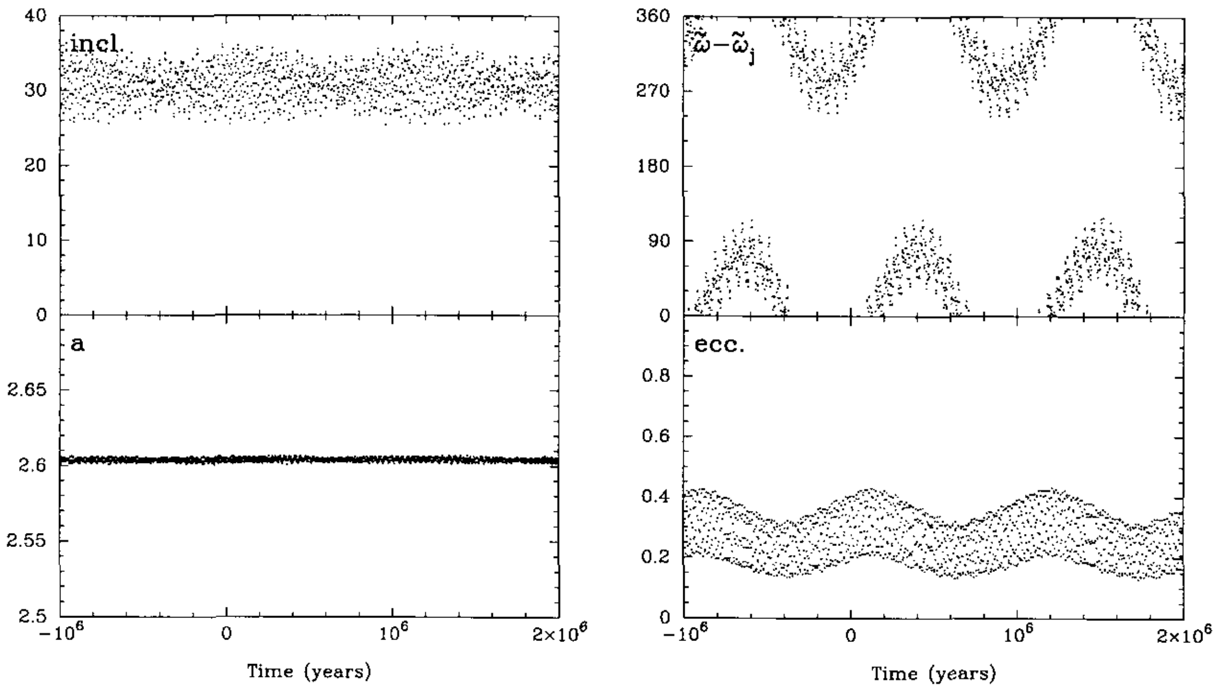


FIG. 8. The evolution of the asteroid (4451) Grieve. The asteroid is locked into the $\nu_5$ resonance (the critical angle $\tilde{\omega}$-$\tilde{\omega}_J$ librates around 0°). However, since in the main asteroid belt the $\nu_5$ resonance is not very strong, the eccentricity exhibits only small long-period oscillations. Conversely, the short periodic oscillation of e is due to the circulation of $\omega$, being the asteroid at large inclination. The asteroid is temporary Mars-crosser.

It is worth discussing in some more detail the cases of the two asteroids (2201) Oljato and (6063) 1984 KB. These two bodies were previously studied over a shorter integration time ($-10^5 < t < +10^5$ years) by Milani *et al.* (1989) and were classified by them as belonging to the so-called Oljato class, characterized by irregular changes in semimajor axis and large eccentricities with secular (increasing or decreasing) trends. Here we have integrated these two orbits with both our numerical integrators. The results obtained by the Radau integrator are those reported in Table I. The results obtained by the Bulirsch–Stoer program are those discussed in Farinella *et al.* (1994) and in Valsecchi *et al.* (1995). (2201) Oljato hits the Sun in both forward numerical integrations; however, the fall time is $t$ = 1.4 Myr with the Radau integrator and only $t$ = 0.36 Myr with the Bulirsch–Stoer one. On the other hand, the evolution of (6063) 1984 KB is qualitatively different in the two numerical integrations, at least after a few hundred thousand years since the starting epoch. According to the Bulirsh–Stoer integration (Valsecchi *et al.* 1995), (6063) 1984 KB collides with the Sun in the backward integration, while in the forward integration it evolves to $a$ <2 AU, where it can survive for more than 1 Myr, safe from secular resonances. According to the Radau results, in the backward integration (6063) 1984 KB approaches Jupiter and evolves to a cometary-like quasi-parabolic orbit, whereas in the forward integration it is captured into the $\nu_6$ resonance and falls into the Sun at $t$ = 0.4 Myr. Such differences between the results obtained with two different numerical integrators are not surprising, taking into account the extremely chaotic nature of these orbits, as we discussed in Section 2.

(ii) The secular resonant arguments $\tilde{\omega}$–$\tilde{\omega}_S$ of the asteroids (1468) Zomba, (2368) Beltrovata, and (3102) 1981 QA oscillate around 180° (see Fig. 4, referring to Zomba) for almost the entire span of our integration. The eccentricities oscillate secularly and keep moderate values. (1468) Zomba is only a Mars-grazer (maximum $e \simeq 0.3$), while (2368) Beltrovata and (3102) 1981 QA, whose maximum eccentricities reach about 0.4 and 0.5, respectively, are Mars-crossers. As pointed out by Morbidelli (1993), these asteroids are located in a very unstable region of the $\nu_6$ resonance, so that slightly different starting values, a different perturbation model, or a particular close approach with a planet can change drastically their behavior, possibly shifting them into the regions of the resonance where the eccentricity can be pumped up to very high values. This happens, for instance, in the integrations discussed by Morbidelli (1993) for (1468) Zomba and by Froeschlé and Scholl (1987) for (2368) Beltrovata.

(iii) Asteroids 1992 SY, 1989 DA, and (3671) Dionysius are alternators with respect to the $\nu_6$ secular resonance (see Fig. 5, referring to 1989 DA). This means that the critical argument $\tilde{\omega}-\tilde{\omega}_S$ alternately circulates over 360° or librates around the stable equilibrium point at 0°, namely depending on the intantaneous value of the semimajor axis. The eccentricities of 1992 SY and 1989 DA vary between about 0.5 and 0.7. Many close encounters with Mars and Earth occur, causing the semimajor axes to change chaotically in a typical random-walk fashion. On the contrary, asteroid (3671) Dionysius quits the secular resonance due to a close encounter, and is subsequently temporarily captured (in the interval between about $1 \times 10^5$ and $3.5 \times 10^5$ years) in the secondary secular resonance involving the critical argument $2\tilde{\omega}-\tilde{\omega}_J-\tilde{\omega}_S$. Under the effect of this secondary resonance, a burst in the eccentricity (up to about 0.82) and one in inclination (up to 30°) occur. At the same time, the asteroid has many close approaches to Earth, and its semimajor axis decreases down to about 1.7 AU (see Fig. 6).

(iv) Five asteroids, namely (512) Taurinensis, (3402) Wisdom, 1991 VC, (3858) 1986 TG, and (4924) Hiltner are located near, but not inside, the $\nu_6$ secular resonance. Their $\tilde{\omega}-\tilde{\omega}_S$ secular arguments circulate regularly forward [see Fig. 7, referring to (512) Taurinensis] and at the same time their eccentricities oscillate secularly reaching maximum values of ≈0.3. They are all Mars-grazers.

*3.1.2. Secular resonance $\nu_5$.* Only one asteroid, (4451) Grieve, is found in the $\nu_5$ secular resonance (see Fig. 8) during the whole integration time span, with the critical argument $\tilde{\omega}-\tilde{\omega}_J$ librating around 0°. The eccentricity and the inclination oscillate secularly between about 0.2 and 0.4 and between 26° and 35°, respectively, so that this asteroid is only a Mars-grazer. This is consistent with the previous results of Scholl and Froeschlé (1990), who concluded that the $\nu_5$ secular resonance is not an efficient mechanism for pumping up the eccentricity.

*3.1.3. Secular resonance $\nu_{16}$.* Two asteroids are strongly affected by the $\nu_{16}$ resonance: (4596) 1981 QB and 1992 RD. Asteroid (4596) 1981 QB has a starting inclination of 37.1°. Its critical argument of the $\nu_{16}$ resonance $\Omega-\Omega_S$ changes very slowly and in an irregular way. At the same time, the argument of perihelion $\omega$ alternates between circulation and libration about 90° or 270°, with a behavior similar to that of (2335) James, as described in Froeschlé *et al.* (1991). This alternating behavior corresponds to temporary captures into the Kozai resonance ($g = s$). As a consequence of the closeness between the $\nu_{16}$ and the Kozai resonances, the inclination grows up to 54°, and the eccentricity can become as large as 0.7. Close encounters with the inner planets, which are quite rare due to the large inclination of the orbit, cause small random changes of the semimajor axis (Fig. 9), because of the relatively high encounter velocity.

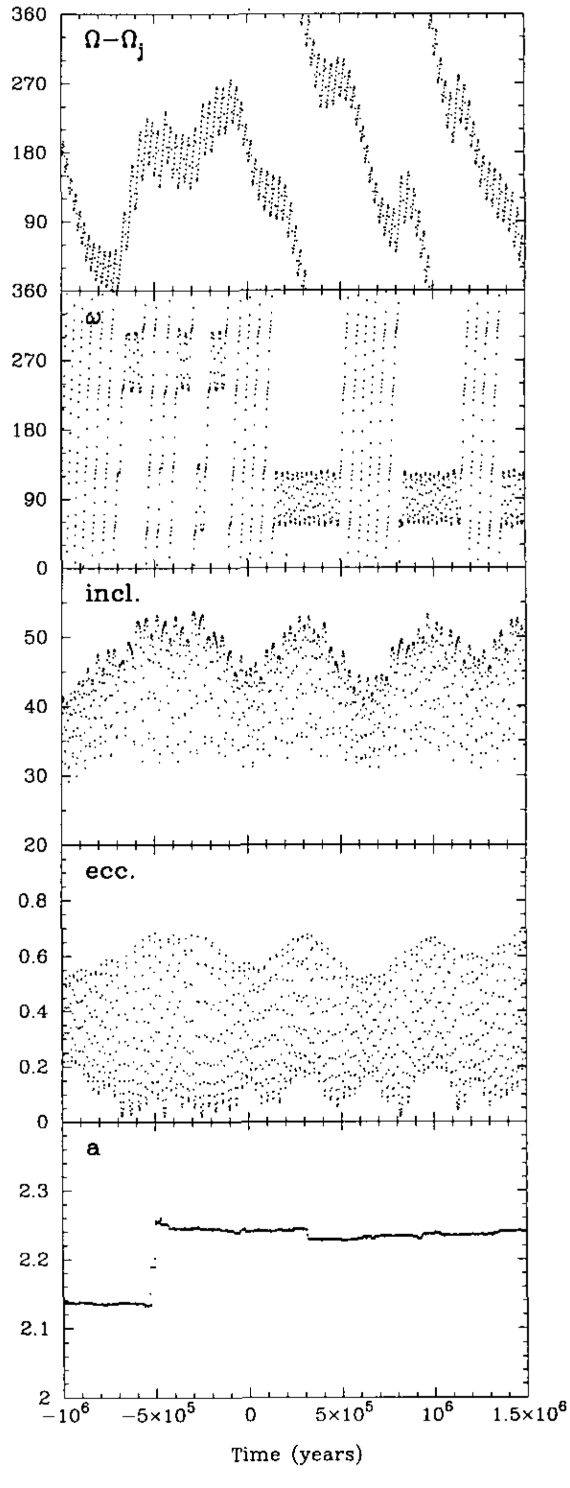


FIG. 9. The evolution of asteroid (4596) 1981 QB. The asteroid is in/close to the $\nu_{16}$ resonance (the critical angle $\Omega$-$\Omega_J$ describes one libration and two slow circulations). During the evolution, the asteroid is temporarily captured into the Kozai resonance, characterized by libration of $\omega$. The big oscillation of e is precisely due to the vicinity of the Kozai resonance. Conversely, $i$ shows large short period changes due to the Kozai resonance modulated by the effect of the $\nu_{16}$ resonance.

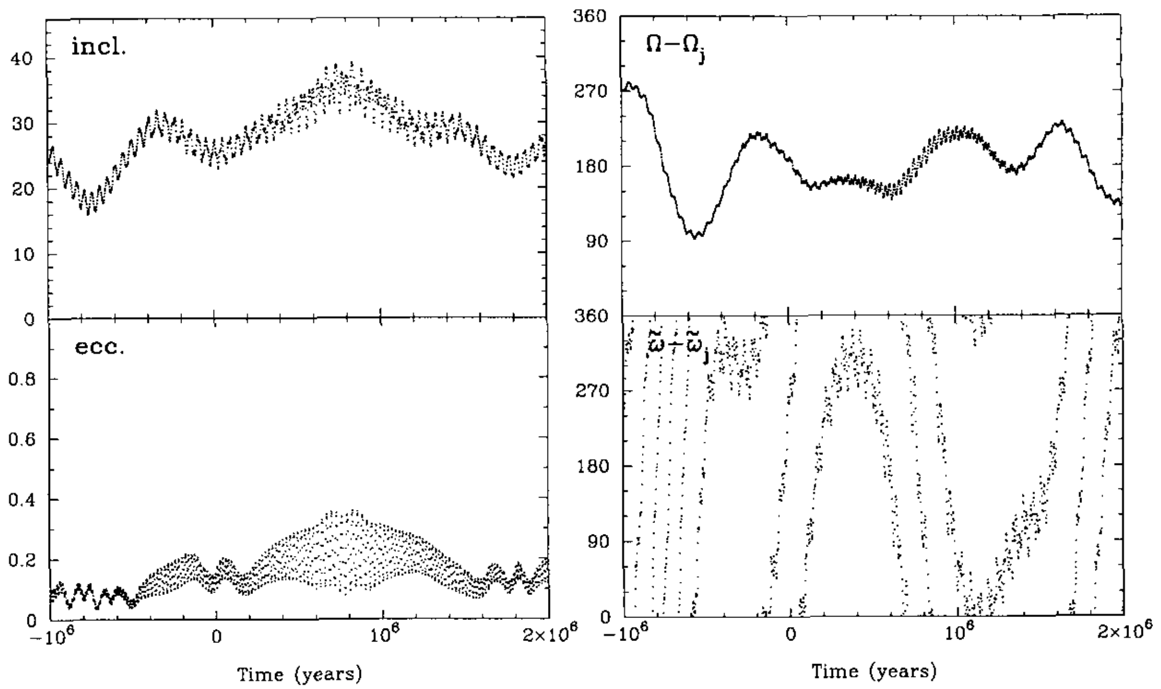


FIG. 10. The evolution of the asteroid 1992 RD. The asteroid is in the $\nu_{16}$ resonance (the critical angle $\Omega$-$\Omega_J$ librates around 180°). Moreover, it is also temporarily in the $\nu_5$ resonance, alternating between circulation and libration of $\tilde{\omega}$-$\tilde{\omega}_J$.

Asteroid 1992 RD is not a Mars-crosser at the present time. However, analyzing its proper frequencies, we realized that it is in the $\nu_{16}$ resonance, so we decided to integrate it as well. The results are illustrated in Fig. 10. The critical argument $\Omega$–$\Omega_S$ of the $\nu_{16}$ resonance librates around 180° during the whole integration span; moreover, during the interval $-6 \times 10^5 \leq t \leq 1.6 \times 10^6$ years, the asteroid is also temporarily located in the $\nu_5$ secular resonance. During this period the eccentricity oscillates between 0.1 and 0.35, otherwise the maximum eccentricity is only 0.15. In a similar way, the peak inclination is about 40° when the asteroid is in both resonances, while it stays at about 26° when the orbit is not in the $\nu_5$ resonance. Thus the asteroid is only temporarily a Mars-grazer.

*3.1.4. Mean motion resonances.* In this work we have not looked systematically for asteroids in mean motion resonances. As explained in Section 2, our selection procedure was rather aimed at MCs and NEAs in or close to secular resonances. However, the orbit of asteroid (5731) 1988 $VP_4$, which was expected to have dynamical instabilities according to the results shown in Fig. 1, turns out to become locked in the 3/1 resonance in a few hundred thousand years. The dynamical evolution of this asteroid, therefore, is dominated by a mean motion resonance, and this is the reason why we devote to it a separate subsection. An exhaustive numerical study of the real NEAs/MCs in jovian mean motion resonances — which form the Alinda class according to Milani *et al.* (1989)—will be the subject of a forthcoming paper.

The orbital evolution of (5731) 1988 $VP_4$ is illustrated in Fig. 11. The asteroid is captured into the 3/1 resonance from $-4 \times 10^5$ to $-2 \times 10^5$ years and from $4.5 \times 10^5$ years onward. Once in the 3/1 resonance, a strongly chaotic evolution of the eccentricity, the inclination, and the semimajor axis is apparent. Such a chaotic behavior is due to the overlapping of secular resonances inside the 3/1 mean motion resonance, as described by Moons and Morbidelli (1995). Indeed, looking at the behavior of the critical angles, we see that this object spends most of the time inside or in the vicinity of the $\nu_6$ resonance and of the Kozai resonance and passes also through the $\nu_5$ resonance. More precisely, the dynamical evolution of (5731) 1988 $VP_4$ is affected by the $\nu_6$ resonance up to $t = 8 \times 10^5$ years; as a consequence, the eccentricity oscillates three times between about 0.6 and 0.9. From $t = 8 \times 10^5$ to $t = 9.5 \times 10^5$ years, the asteroid is influenced by the Kozai resonance; the eccentricity is decreased down to almost zero, but the inclination is pumped up to 50°. Still in the Kozai resonance, the eccentricity is then pumped up again to 0.6 and the inclination down to 20°. Then, starting at about $t = 9.5 \times 10^5$ years, the asteroid stays very close to the $\nu_6$ resonance; the eccentricity is pumped up to 0.95 and, eventually, to nearly unity, causing the object to fall into the Sun. It is interesting to

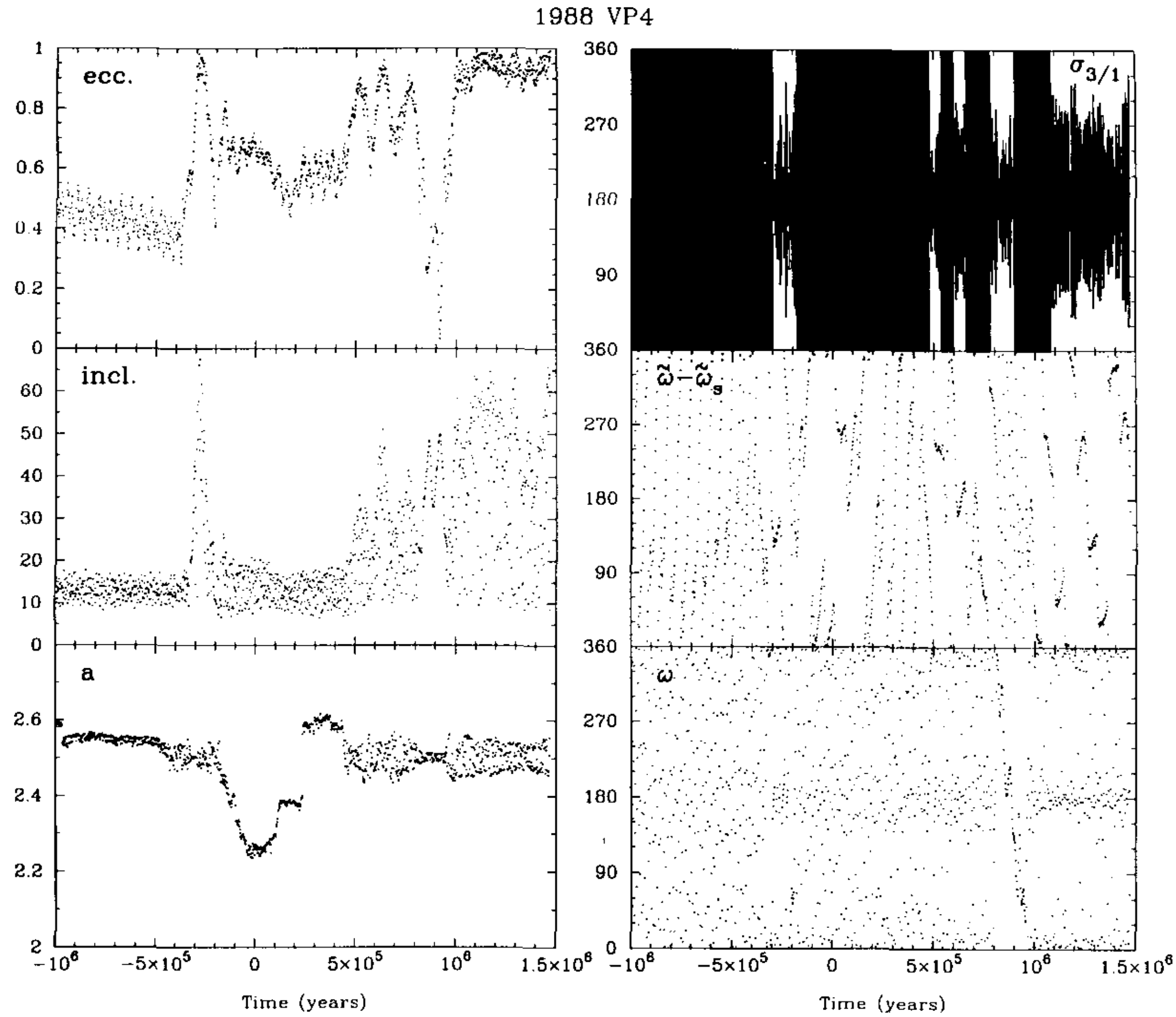


FIG. 11. The evolution of the asteroid (5731) 1988 VP$_4$. The asteroid is captured into the 3/1 resonance between $-4\times 10^5$y $< t < -2\times 10^5$y and from t= $4\times 10^5$y onwards. Once in the 3/1 resonance, the eccentricity and the inclination change wildly, due to the overlapping of secular resonances inside the 3/1 commensurability.

note that the asteroid needs just about 1 Myr from the time of capture into the 3/1 commensurability to its end into the Sun.

### 3.2. Asteroids with Semimajor Axis <2 AU

As explained in Section 2, we have integrated eight NEAs having starting semimajor axes in the range 1.24 AU $\leq a \leq$ 1.93 AU. Most of them have initial eccentricities <0.6 and inclinations <10° (see Table I). The main reason for doing this is that the location of secular resonances and their dynamical effects have been studied in detail in the main belt, i.e., for $a$ >2 AU (Williams and Faulkner 1981; Morbidelli and Henrard 1991a,b; Knežević *et al.* 1991), but no comparable analytical work has been done so far in the region $a$ <2 AU. On the other hand, all the numerical integrations concerning this zone discussed in the published literature cover time spans which are too short for detecting the existence of secular resonances and finding out their effects.

Our numerical integrations show for the first time that secular resonances are active also inside the inner edge of the main belt. In particular, we have found that: (i) the $\nu_5$ resonance appears to be located at moderate inclinations in this region (while in the main belt it can be found only at inclinations ≈30°), so this resonance probably plays a significant role in the orbital evolution of many NEAs/MCs; (ii) the $\nu_{16}$ resonance exists also for $a$ <2 AU and is active for five of the eight orbits that we have integrated.

Coming to a detailed discussion of our results, we can distinguish three kinds of orbital behavior.

*3.2.1. Orbits strongly affected by secular resonances.* The orbits of the asteroids (1862) Apollo, (1864) Daedalus, and (2101) Adonis are strongly affected

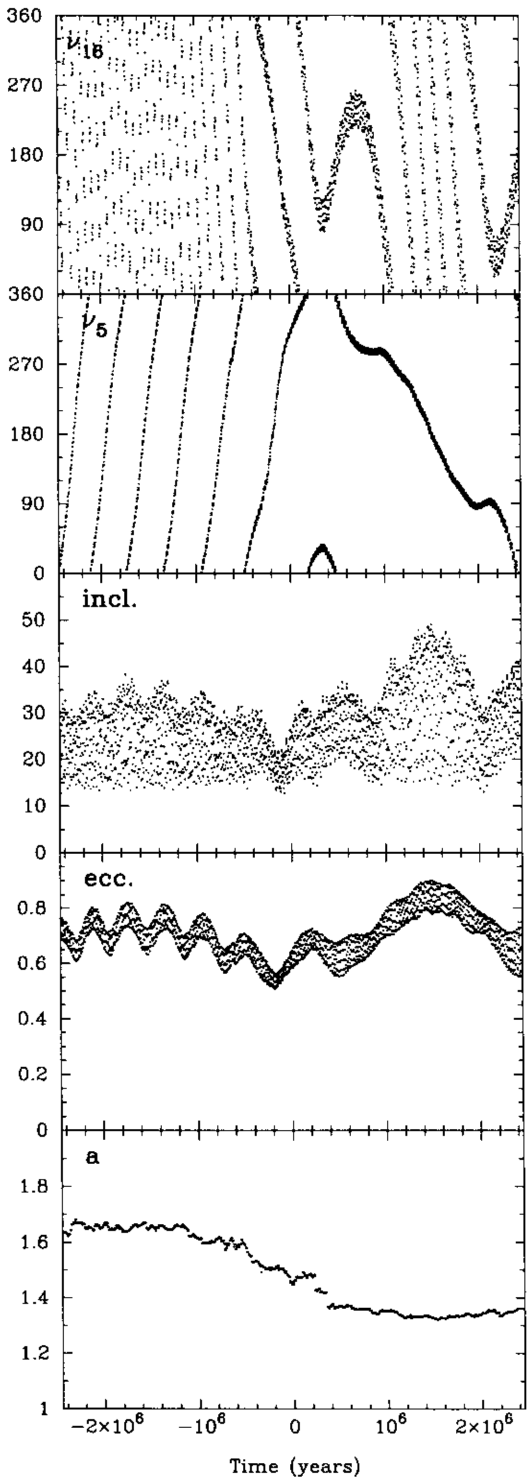


FIG. 12. The evolution of the asteroid (1864) Daedalus. The asteroid is temporarily in the $\nu_5$ and in the $\nu_{16}$ resonances. This is the first time that we point out the presence of these two resonances in the inner Solar System (a<1.8 AU).

by the $\nu_5$ and/or the $\nu_{16}$ resonances. When the critical argument $\tilde{\omega}-\tilde{\omega}_J$ of (1862) Apollo librates around 0° (from $t \simeq 8\times 10^5$ years until the end of the integration time span), the inclination and the eccentricity increase regularly up to about 28° and 0.8, respectively. (1864) Daedalus (see Fig. 12) is alternately in the $\nu_5$ and $\nu_{16}$ resonances or simultaneously in both of them (for 0.2 Myr $< t <$ 1 Myr). During its stay in the overlapping region, the inclination suffers large oscillations between about 12° and 49°, and the eccentricity varies from 0.80 to 0.86. Asteroid (2101) Adonis—which has osculating elements similar to those of the Taurid meteoroid complex (see Asher *et al.* 1993, and Štohl and Porubčan 1992)—is never in a secular resonance until $t$ = 0.6 Myr, and then it is injected into the $\nu_5$ resonance by a close planetary encounter. Subsequently, the eccentricity undergoes moderate oscillations up to almost unity, a solar collision occurring at $t$ = 0.7 Myr. In Farinella *et al.* (1994), where the integration was performed with the Bulirsch–Stoer integrator instead of the Radau one, the solar collision occurred at $t$ = 0.9 Myr, and the overall behavior of the orbit was qualitatively the same. In Milani *et al.* (1989), (1862) Apollo and (1864) Daedalus were classified in the so-called Geographos class, characterized by secular trends in eccentricity and inclination and by frequent close Earth approaches, whereas (2101) Adonis was classified as a member of the strongly chaotic Oljato class, like (2201) Oljato itself and (6063) 1984 KB.

*3.2.2. Orbits dominated by close encounters.* Three asteroids, namely (3757) 1982 XB, (3908) 1980 PA, and 1977 VA, have an orbital evolution essentially dominated by close encounters and are not in any secular resonance [see Fig. 13, referring to (3908) 1980 PA]. Also in these cases the orbital evolution was previously studied by Milani *et al.* (1989). While (3908) 1980 PA was classified as a Geographos class NEA, 1977 VA and (3757) 1982 XB were classified as members of the Eros class, characterized by fairly small changes in semimajor axis (as their orbits are Mars- but not Earth-crossing) and significant secular trends in eccentricity and inclination.

*3.2.3. Puzzling cases.* The orbital evolutions of (1620) Geographos and (2135) Aristaeus are somewhat strange and puzzling for us. These two asteroids were assigned to the Geographos class in Milani *et al.* (1989). Here, both orbits seem to be related to the $\nu_5$ and $\nu_{16}$ resonances, as the corresponding critical angles $\tilde{\omega}-\tilde{\omega}_J$ and $\Omega-\Omega_S$ change slowly and irregularly. However, neither the eccentricity nor the inclination changes appear to be correlated to the behavior of the critical angles of the $\nu_5$ and $\nu_{16}$ secular resonances. Secular perturbations due to the inner planets are probably effective here, and further work is needed to identify

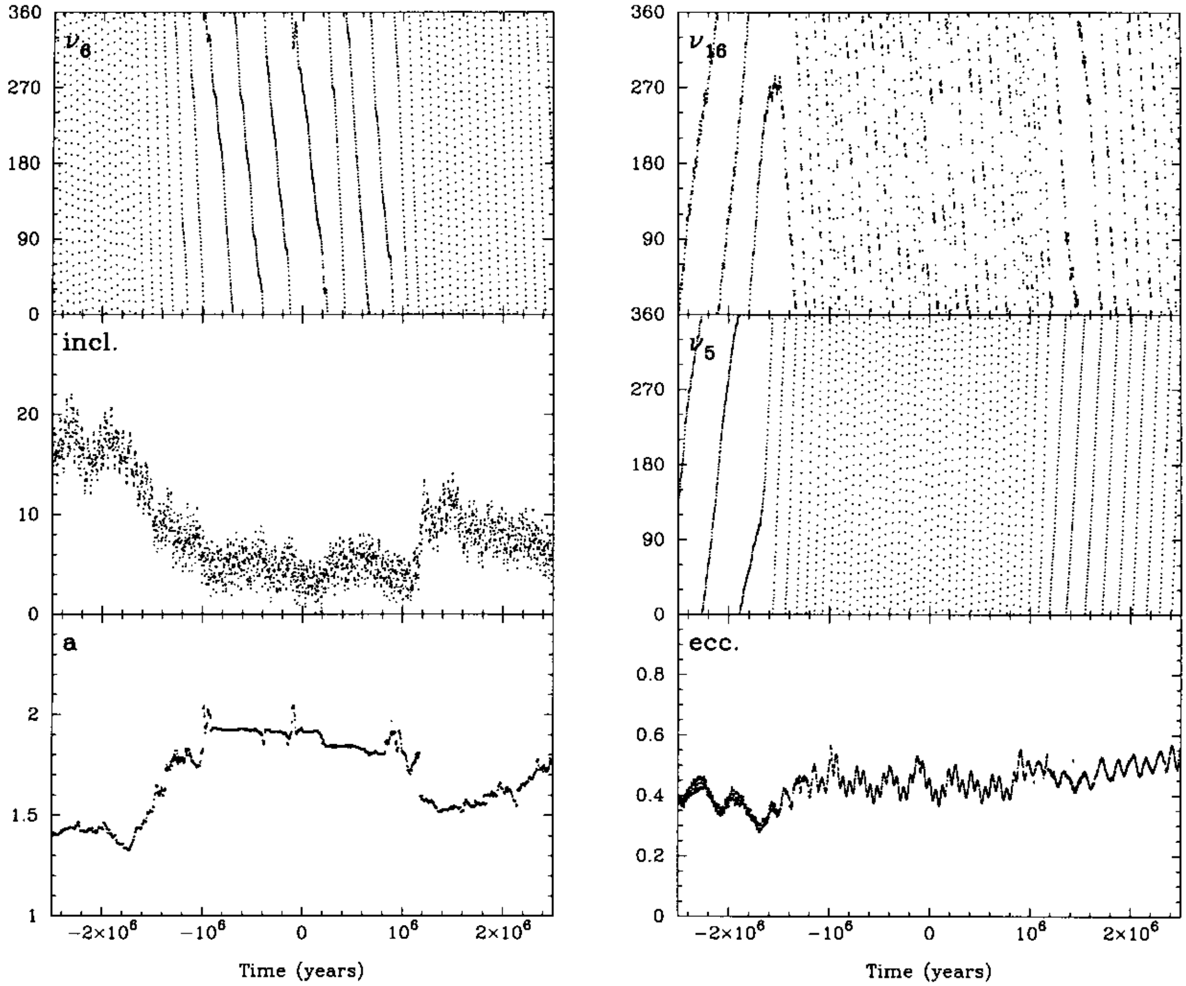


FIG. 13. The evolution of the asteroid (3908) 1980 PA. The evolution is dominated by close encounters. We remark the passage through the $\nu_{16}$ resonance.

and understand the specific dynamical mechanisms at work in these cases (see Fig. 14).

## CONCLUSIONS

In this paper we have selected the 24 NEA and MC objects with $a$ >2 AU, which were expected to be close to one of the three main secular resonances, according to our computer-assisted analytical theory. Then we have integrated numerically the orbits of these bodies, confirming their resonant behavior. In this way we have shown that the $\nu_6$ resonance affects strongly the dynamics of several planet-crossing asteroids with large eccentricities. This leads us to speculate that this resonance can play an important role in the evolution of many other NEAs (the currently "slow-track" ones, see below). On the other hand, the $\nu_5$ and $\nu_{16}$ resonances influence only a few bodies and have much smaller effects, confirming the conclusions of Morbidelli (1993) and Scholl and Froeschlé (1986, 1990).

The results of our numerical integrations can be summarized by stating that the dynamical evolution of NEAs and MCs is characterized by the existence of dynamical *fast tracks* and *slow tracks*:

- Fast tracks are due to resonances (the $\nu_6$ secular resonance and some jovian mean motion resonances, such as the 3/1 and 5/2 ones, for the latter we refer to Farinella *et al.* 1994 and Morbidelli and Moons 1995), which pump the eccentricity up and down in a very efficient and quick manner. Therefore, on a time scale of the order of 1 Myr, main-belt asteroids in such fast tracks can enter and exit the planet-crossing region (defined here by the lower limit $e$ > 0.3), eventually to collide with the Sun (whenever the eccentricity is increased to almost unity) or to encounter Jupiter and be ejected on a

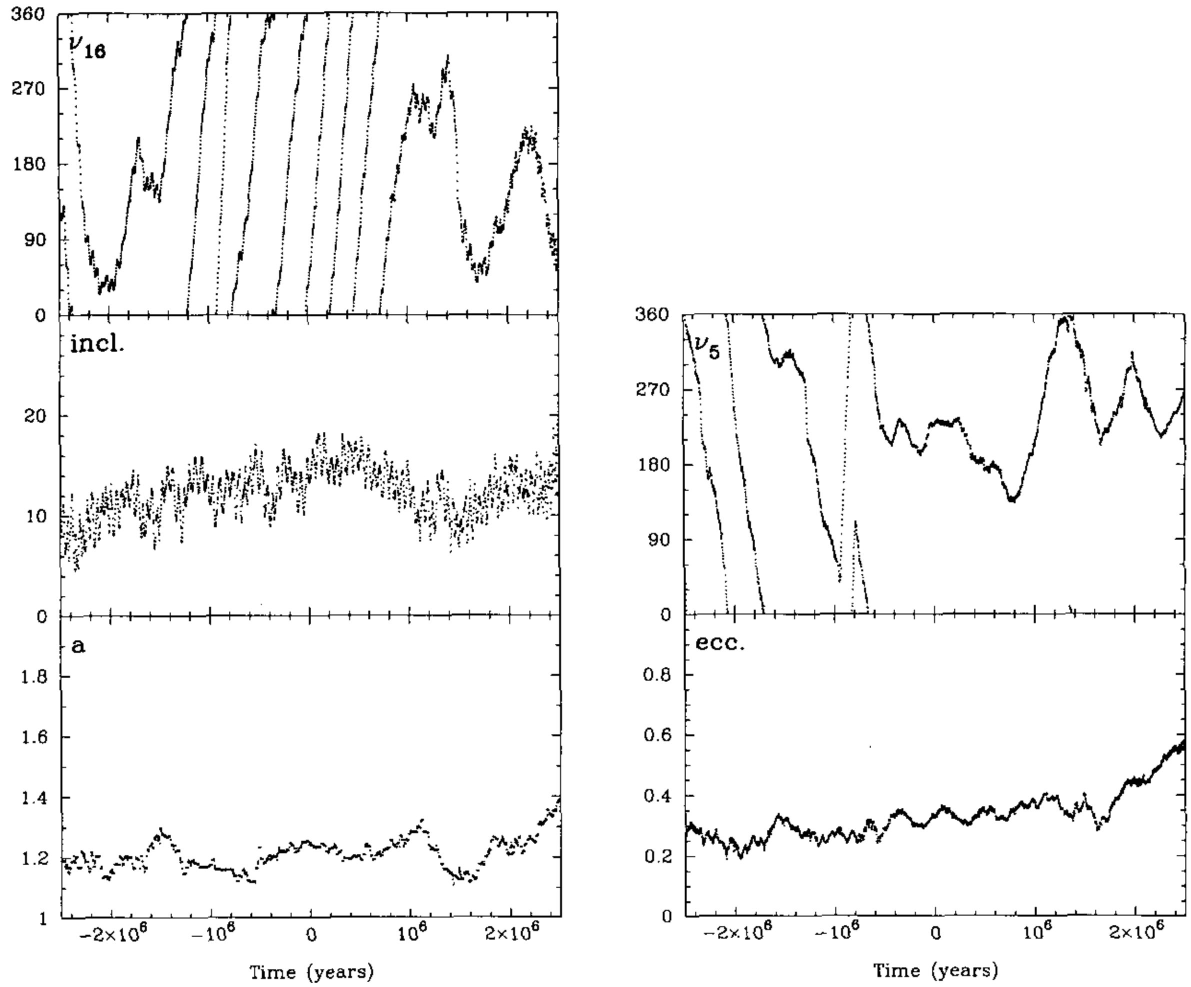


FIG. 14. The evolution of the asteroid (1620) Geographos. The asteroid is temporary in the $\nu_5$ and $\nu_{16}$ resonances, according to the behaviour of the critical angles $\tilde{\omega}$-$\tilde{\omega}_J$ and $\Omega$-$\Omega_J$. However the behaviour of the eccentricity and of the inclination is very irregular without any definitive corelation with the evolution of the critical angles.

hyperbolic orbit (when the semimajor axis exceeds about 2.5 AU). Our results suggest that hyperbolic ejections are somewhat less frequent than solar collisions.

- Slow-track evolutions occur outside resonances. Here the dynamics is dominated by chance close encounters with the inner planets, which gradually change the asteroid orbital elements. In particular, the semimajor axis varies in a random walk manner, very slowly when only Mars is approached and somewhat faster when Earth and Venus can also be reached (for order of magnitude estimates of corresponding time scales, see Greenberg and Nolan 1989). No strong secular trend characterizes the evolution of the eccentricity and the inclination. Apparently, asteroids can stay in these slow-track states for several million years (see Valsecchi *et al.* 1995, and Jopek *et al.* 1995). However, sooner or later, owing to random changes in semimajor axis caused by close encounters, they enter into a resonance and start to evolve along a fast track. Then, their lifetime is short (as discussed above), unless a new strong close encounter with a planet changes again the semimajor axis, extracting them from the fast track before they are lost.

This is for the moment just a qualitative picture, and many questions await new numerical work for a quantitative answer. For instance: how many different "dynamical classes" are required to correctly classify the evolutionary patterns over time spans of 10 to 100 Myr, of the same order as the collisional lifetimes of NEAs and the cosmic-ray exposure ages of many meteorites? Do secondary secular resonances play any role on such long time scales? How frequently do exchanges occur among different classes, and more in general between fast-track and slow-track orbits with very different evolutionary speeds? How effective are the mean motion resonances with Earth and the other inner planets in protecting NEAs from close encounters, as in the case of the Toro-class objects of Milani *et al.* (1989)? The work on these unsolved problems, on the other hand, will have to be based on some general conclusions obtained in this paper, such as that it is probably meaningless to talk about a single "typical dynamical lifetime" for NEAs and/or MCs, and that the demography and evolution of this population of bodies is certainly more diverse and complex than it has been assumed so far. In particular, the role of secular resonances cannot be neglected, nor should it be treated in an oversimplified way lest the real relationships between slow and fast tracks are lost in the models.

Another interesting result of this work comes from our integrations of NEAs with current semimajor axes <2 AU. Since no analytical theory can properly take into account the effects of the inner planets on these orbits, as explained at the end of Section 2, we chose our sample among the orbits integrated on a shorter time span by Milani *et al.* (1989), and displaying already secular trends in the eccentricity and/or inclination. We have shown that the $\nu_5$ and $\nu_{16}$ resonances are present also in the region well inside the inner edge of the main belt, a result which cannot be recovered with analytical theories including the effects of the outer planets only. We plan to integrate more orbits of this kind in the future, in order to improve the statistics and to better understand the dynamical mechanisms at work.

## ACKNOWLEDGMENTS

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