Reminders: ELEMENTS OF FOURIER OPTICS, volume 1,

Yves Rabbia, Obs Côte d'Azur provisional course notes, to complete, to correct,...

1. Representation of the "light" phenomenon

Some experiments invite to use a wave model (interferences, diffraction), other experiments favor a corpuscular description (emission and absorption of photons, photoelectric effect). The resolution of the problem posed by the modeling of the "light" phenomenon belongs to quantum mechanics and we won't worry about it. In other words, we will not be interested in the question: “How do we go from the emission of photons by a set of atoms to the existence of a wave surface which unfolds in space surrounding ?”
Our model of the "light" phenomenon will be here that of the oscillating electromagnetic field, its behavior being governed by Maxwell's equations.
We are led to represent the "light" phenomenon by the trihedron electromagnetic ($\vec{P}$, $\vec{E}$, $\vec{H}$) where $\vec{P}$ is the Poynting vector describing the direction of propagation, $\vec{E}$ is the electric field and $\vec{H}$ the magnetic field , theycarry information on the power transported and on the polarization of the field.
To convey the behavior of light waves, it is customary to consider simply the vector $\vec{E}$.
Immediately we introduce a restriction of the problem: the scalar approximation
The field will be for us a scalar and not a vector. We will therefore generally ignore the polarization aspect. This restriction is of no consequence in the case of beams encountering surfaces at low incidence, except perhaps in the case of diffraction gratings. Even in the latter case we will remain in the approximation scalar, unless explicitly stated.
Considering a monochromatic wave of wavelength "λ", a common analytical representation for the fields "E" at time "t" at a point P0 in space is that of a sinusoid such that:

$$E0(t) = A.\cos(2\pi.\nu.t - \varphi o)$$

where A is the amplitude of the oscillation, "ν" is the frequency of the oscillation ($\nu = c/\lambda$), and "φ0" the phase at time origin.
It is usual to evaluate by $|E|^2$ the power transported by the wave.
At a point P on the axis of propagation and such that the distance d(P0,P) = r , the expression of the field will be:

$$E(t) = A.\cos[2\pi.\nu.(t - \Delta t) - \varphi o]$$

in which "Δt" represents the time taken by the disturbance at point P0 to reach the point P. Here we have $\Delta t = r/v$ where "v" is the speed of propagation, (which is "c" in vacuum and “c/n” in a medium of optical index "n").
We therefore have $\Delta t= r/v = n.r/c$

$$E(t) = A.\cos[2\pi.\nu .(t - (r / v)) - \varphi 0]= A.\cos[2\pi.\nu.(t - (n.r/ c)) - \varphi 0]$$

The quantity “n.r” is called “optical path”. The illustration of this formalism is given in the complement on propagation.
Let us note that $2\pi.\nu.n.r/c = 2\pi.n .r/ \lambda$ since $\nu = c/\lambda$

Remarks :
a). At the cost of mathematical precautions that we leave aside here, the use of complex exponentials gives great flexibility of calculation (analytical signal, Hilbert transform). We will come back to the difficulties of this choice. from now and on we will prefer to describe the wave by E(t) which expression is:

$$E(t) = A.\exp(i.2\pi.\nu.t - \varphi0 - (2\pi/\lambda).n.r) = A.\exp(- i. 2\pi.n.r/\lambda).\exp(i.2\pi.\nu.t - \varphi0).$$

b). Most often we will only have to consider the propagation in a medium of index n = 1.

c). It is customary to call "complex amplitude" the quantity $A.\exp(- i. 2\pi\ \lambda. n.r)$ It describes the wave spatially. We will elaborate on this point later.

d). For ease of writing, we agree to replace the sign "-" by the sign "+" which for our subject is without inconvenience.

e). The phase at the origin is important when one is interested in the coherence of light and the modeling of the radiation emitted, but for our subject this aspect will be managed implicitly and we can without inconvenience consider it to be zero. Considering these remarks, the wave can be described by:

$$E(t) = [A.\exp(i . 2\pi.r/ \lambda)]. \exp(i.2\pi.\nu.t)$$

f). In a real physical situation, the measurable energy results from a temporal integration (response of the detector), an integration over an elementary spectral interval (the concept of a monochromatic wave is an idealized notion), of an integration on a surface element (the detector of extent reduced to a point does not exist) and an integration on a solid corner element (isolating a specific direction is an idealized notion). Therefore when we talk about intensity | E |2 we are talking about an energy per unit of time, per unit of frequency, per unit area, per unit solid angle and an energy W actually collected is described by:

$$W= \int_{\Delta t\ \Delta\nu\ S\ \Omega} | E |^2\mu.\ dt.d\nu.d\sigma.d\varpi$$

(the notation ∫ is used for "integration", this a defect of the equation translator)

This question will only very rarely concern us and will be studied in another course module. We will then recognize in | E |2 what in photometry is called "specific intensity". However, be aware of this point. It is expressed for example in W / (m2.Hz.Steradian).

From a slightly more general point of view (pretending to ignore the formalism given above), we describe the light wave by $u(x,y,z,t,\lambda)$ resulting from the processing of Maxwell's equations in vacuum (Differential equations). It is shown in good books that this treatment results in two representations of the electromagnetic disturbance $u(x,y,z,t,\lambda)$ which we call monochromatic light wave of wavelength λ: that are the spherical wave and the plane wave.

Spherical wave

reaching point P(x,y,z) and coming from point P0 (x0, y0, z0) with $r_{(P0,P)}$ for the distance from P to P0:

$$u(P, \lambda, t) = [1/ r_{(P,P0)}] . U(t,\lambda). \exp(i.2\pi.\nu.t) . \exp[- i (2\pi/ \lambda) .r_{(P,P0)}]$$

Plane wave
propagating along the z axis (coming from a pointlike source at infinity) reaching the point P(0,0,z):

$$u(z, \lambda, t) = U(t, \lambda). \exp(i.2\pi.\nu.t) \exp(i\ 2\pi.z/ \lambda)$$

In the first case, the factor $1/r(P,P0)$ expresses the dilution of the initial energy over the whole of the wavefront , which, in the second case does not manifest itself (the specific intensity is independent of the distance of the source).

Let us note that the time dependence of $U(t, \lambda)$ is significantly slower (typically 100000 times) than that of exp $i2\pi\nu t$. We will ignore this dependency here.

So, for a given wavelength, the two respective situations will therefore be described by

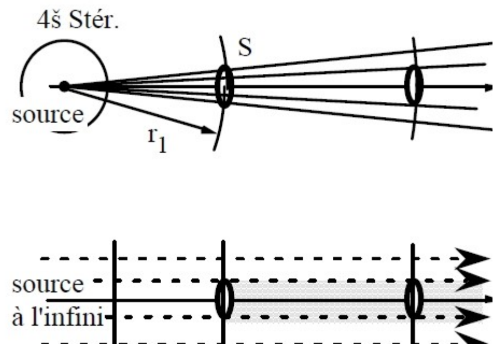
$$u(P, t) = [1/r(P;P0)] . [U.\exp(i. (2\pi.r(P,P0) / \lambda)]. \exp(i.2\pi.\nu.t)$$

$$u(P, t) = [U. \exp(i. (2\pi.z/ \lambda)] . \exp(i.2\pi.\nu.t)$$

To lighten the writing we denote by U(P) the complex amplitude (in English: phasor)
$U.\exp(i. 2\pi. r(P,P0) / \lambda .)$.

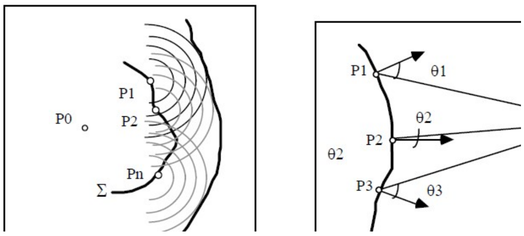
Wavefront

This is the surface formed by the points P reached by the wave with the same phase.
For a plane wave, we have a plane as the wavefront results from z = constant.
For a spherical wave coming from $P0(x0,y0,z0)$ we have
$r(P0,P)$ = squareroot of $[(x-x0)^2+(y-y0)^2+(z-z0)^2]$ = constant,
which defines a sphere centered at P0.

With the spherical wave, the collecting surface S receives the radiation in the solid angle $\omega1$; $\omega1 = S/(r1)^2$ and takes the fraction "$\omega1/ 4\pi$" of the total energy emitted. If S is more far away it takes a lesser fraction. With the plane wave, it takes the same fraction of the propagated energy regardless of the distance from the source. There is no need to considere dilution of energy.	

2. *Principle of Huyghens-Fresnel and application to diffraction*

We faced the following problem: Knowing the monochromatic complex amplitude distribution at any point P of a wavefront Σ (equiphase surface) what is the complex amplitude at a distant point Q? The basic concept that will guide us is the following: Each point P on the surface Σ acts as a pointlike source emitting a spherical wave. These waves (wavelets) are synchronous (in phase).
The complex amplitude at point Q is the sum of the complex amplitudes received at this point and exits from the different points P.
This is , in short, the main meaning of the principle of Huyghens Fresnel.



In the case of a wave encountering a plane screen perpendicular to the direction of propagation, and whose transmission is zero except on a surface S called diffracting pupil where the transmission is 1, a formalism developed by Kirchhoff and Helmholtz, allows to give a general expression of the amplitude in Q:

$$U(Q) = (1/i\lambda).\int_S [1/r(Q;P)].\ U(P)\ .\ K(\theta)\ .\ \exp(i.\ 2\pi\ r(Q;P))/\ \lambda).\ d\sigma$$

where U(P) is the complex amplitude at point P (running point on S) and where K(θ) represents a kind of indicatrix emission, varying between 1 and 0 when θ increases (i.e. when PQ deviates from the normal to the wave surface). For more details see Goodman: Introduction to Fourier Optics.

Note: "S" is not necessarily a portion of wavefront, but it is necessary to know the distributon of the complex amplitude all over this surface "S".

Under certain conditions, the previous expression simplifies: these are the Fresnel conditions and the conditions of paraxial optics:

Fresnel conditions:
typical pupil size and PQ distance large compared to the wavelength.
Paraxial conditions: lightly open beams (PQ>>pupil dimension) and lightly inclined rays on the axis (low θ).

With these approximations the Kirchhoff-Helmholtz expression is simplified.
We locate by Oz the direction of propagation, the coordinates of a point of the diffracting plane located in z = 0, are ξ and η (this concerns the points P) and the coordinates in a plane perpendicular to Oz at the abscissa z are x and y (this concerns the point Q or any other point of this plane where one wants to know the complex amplitude). We then have:

$$U(x,y) = (1/i\lambda) \int_S [1/\ r(x;y;\xi;\eta)]\ \Pi(\xi;\eta).K(\theta).\exp(i.\ 2\pi.\ r(\xi;\eta;x;y)/\ \lambda.).\ d\xi.d\eta$$

where r(x,y,ξ,η) is the PQ distance described by

$$r^2(x,y,\xi,\eta) = z^2 + (x - \xi)^2 + (y - \eta)^2$$

We can recognize this expression as resulting from the Pythagore's theorem

Another formulation (likely to be more human) is :

$$U(x,y) = (\text{KH factor}) \int_{(\text{pupil surface})} [\text{dilution}]\ (\text{pupil transm}).(\text{obliquity}).(\text{complex amplit})\ d\xi.d\eta$$

The previous conditions allow the following approximations.
The factor 1/r(x,y,ξ,η) in the integrand (dilution) can be approximated by z (largely greater than x,y,ξ,η) and thus comes out of the integral.

In the exponential we must keep the dependence in x and y, but since we have z much larger than transverse dimensions,, we can write:

$$r^2(x,y,\xi,\eta) \approx z^2 .(1+ [(x - \xi)^2 + (y - \eta)^2]/z^2) = z^2 .(1 + \text{something verysmall})$$

Now remembering that
squareroot of (1+ very small) is approximated by (1+(very small)/2)
we can write

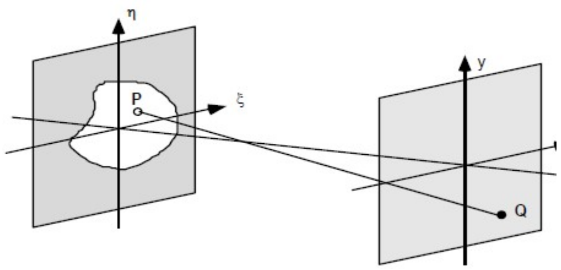
$$r(x,y,\xi,\eta) \approx z.(1 + [(x - \xi)^2 + (y - \eta)^2]/2z^2) = z+ [(x - \xi)^2 + (y - \eta)^2]/2z)$$

It is necessary to keep this dependence because the variation of the argument of the exponential is governed not by the difference between r and z, which evolves slowly (paraxial conditions), but by the ratio $(r-z)/\lambda$ who him; evolves very quickly.
On the other hand the angles θ remain low and therefore $K(\theta) \approx 1$ for any point P of the pupil. As a result we arrive at:

$$U(x,y) =(1/ i\lambda z)\int_S U(\xi;\eta).\exp(i.\ 2\pi\ r\ (\xi;\eta;x;y)/ \lambda).\ d\xi.d\eta$$

or by introducing $\Pi(\xi,\eta)$ to describe the transmission of the diffracting pupil (which can be a complex transmission) and expliciting $r(x,y,\xi,\eta)$:

$$U(x,y) = [\exp(i.2\pi.z/\lambda) / (i\lambda z)] . \int_{-\infty}^{+\infty} \Pi(\xi;\eta).U(\xi;\eta).\exp(i.\pi. [(x - \xi)^2 + (y - \eta)^2]/ \lambda z).\ d\xi.d\eta$$



The interpretation of this expression is as follows:
To build the amplitude in (x,y), each wave coming from a point (ξ,η) of the pupil will bring its contribution $U(\xi,\eta)$, but yet keeping memory of the path it traveled from (ξ,η) to (x,y), i.e. taking into account a phase supplement calculated by

$$\varphi(x,y,\xi,\eta) = [2\pi \text{ (optical path)}/ \lambda] = [2\pi .\text{(geometric path).(optical index)}/ \lambda$$

We decided that the optical index of work would be 1, except where mentioned.
If the phase in (ξ,η) is $\psi(\xi,\eta)$, the complex amplitude brought in Q(x,y) by $P(\xi,\eta)$ will have for phase: $\Phi(x,y,\xi,\eta) = \psi(\xi,\eta) + \varphi(x,y;\xi;\eta)$
The phase φ has a part in $\exp(i\ 2\pi\ z/ \lambda)$ independent of the points (ξ,η) and (x,y). This stable phase reports on the "ageing" of the phase simply when the wave travels the distance "z" on the axis of propagation, between the diffracting screen (ξ,η) and the observation plane (x,y). This global phase is common to all points of the pupil and can so get out of the integral.

The other part, depending on the point P (ξ,η) and the point Q (x,y) will fix the relative phases of the contributions arriving at (x,y). They will play a decisive role in the addition of the amplitudes resulting from the points (ξ,η) to form the resulting amplitude at Q(x,y) .
The squared modulus of this sum will give the intensity at point (x,y). On the other hand the phase $2\pi.z/ \lambda$ will disappear in the intensity calculation.

Concerning the transmission $\Pi(\xi,\eta)$, essentially two cases arise:
1. Real transmission of the aperture (single hole with given contours, or several holes)
2. Complex transmission (screen formed of a transparent medium of variable thickness, or surface reflective with flatness defects). The transmission modulus is in all cases between 0 and unity. (hypothtic screens that could cause amplification are ignored).
The complex character is introduced by the action of the screen on the phase of the wave passing through it. This action is based on the optical path traveled between entering and exiting the screen. Generally speaking, given a complex amplitude $A \exp i\varphi(\xi,\eta)$ reaching a screen whose transmission is $P(\xi,\eta).\exp i\psi(\xi,\eta)$, the complex amplitude transmitted by the point (ξ,η) will be: $A.P(\xi,\eta).\exp i(\varphi(\xi,\eta) + \psi(\xi,\eta))$.
This amplitude propagates to the point (x,y) with its phase increasing by $2\pi.r(P,Q)/\lambda$.

3. *Fresnel diffraction*

Let's go back to the expression of the complex amplitude in (x,y)

$$U(x,y) = [\exp(i.2\pi.z/\lambda)]/(i\lambda z) . \int_{-\infty}^{+\infty} \Pi(\xi;\eta).U(\xi;\eta).\exp(i.\pi.[(x-\xi)^2 + (y-\eta)^2]/\lambda z).\, d\xi.d\eta$$

We can explain it by developing the argument of the exponential in the integrand, what writes:

$$\int_{-\infty}^{+\infty} \Pi(\xi;\eta).U(\xi;\eta).\exp((i.\pi/\lambda z))(x^2 + \xi^2 + y^2 + \eta^2 - 2x\xi - 2y\eta).\, d\xi.d\eta$$

By setting $2\pi.z/\lambda = \varphi$, and $\Pi(\xi,\eta).U(\xi,\eta) = f(\xi;\eta)$ we obtain:

$$U(x,y) = \exp(i.\varphi)/(i.\lambda.z).(\exp(i.\pi(x^2 + y^2)/(\lambda.z)).$$
$$\int_{-\infty}^{+\infty} f(\xi;\eta).\exp(i.\pi(\xi^2 + \eta^2)/\lambda.z).\exp((i.2\pi/\lambda z))(2x\xi + 2y\eta)..d\xi.d\eta$$

also written

$$U(x,y) = \exp(i.\varphi)/(i.\lambda.z).(\exp(i.\pi(x^2 + y^2)/(\lambda.z)).$$
$$\int_{-\infty}^{+\infty} f(\xi;\eta).\exp(i.\pi(\xi^2 + \eta^2)/\lambda.z).\exp[-i.(2\pi/\lambda z))(x\xi + y\eta)].d\xi.d\eta$$

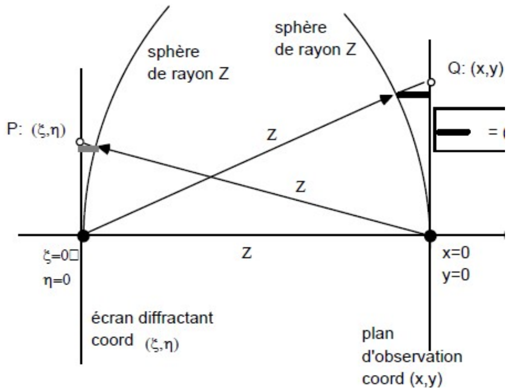
and introducing the variables $u = x/\lambda.z$ and $v = y/\lambda.z$ we have

$$U(x,y) = \exp(i.\varphi)/(i.\lambda.z).(\exp(i.\pi(x^2 + y^2)/(\lambda.z)).$$
$$\int_{-\infty}^{+\infty} f(\xi;\eta).\exp(i.\pi(\xi^2 + \eta^2)/\lambda.z).\exp[-i.2\pi(\xi.u + \eta.v)].d\xi.d\eta$$

expression where we recognize the two-dimensional FT of the function

$$f(\xi,\eta).\exp(i.\pi\,\lambda.z(\xi^2 + \eta^2))$$

this FT being calculated for particular values $u = x/\lambda.z$ and $v = y/\lambda.z$
Here is a powerful way to calculate the amplitude at point (x,y).
But what do these complex exponential factors mean?
Just as $\exp i\varphi$; the factor $\exp(i.\pi(x^2 + y^2)/\lambda.z)$ will disappear in the intensity calculation. One can, as an exercise, reflect on its physical significance by means of the following figure, which does not call for any particular comments.
Basically it consist of taking into account extra optical paths which stand between a sphere and its tangent plane



Another exploitation of the expression of U(x,y) is to recognize in it a **convolution** of the complex amplitude f(ξ,η) transmitted at the point (ξ,η) and the complex exponential under the integrant. We will therefore write:

$$U(x,y) = f(\mathbf{x},\mathbf{y}) \ast \left[\frac{\exp(i\frac{2\pi}{\lambda}.z)}{i.\lambda.z} \; . \; \exp(\frac{i.\pi}{\lambda.z}(\mathbf{x}^2+\mathbf{y}^2)) \right]$$

We can interpret this writing in terms of an operator:
we pass from the complex amplitude transmitted by a screen in a given plane, to the complex amplitude in a plane distant by "z" on the direction of propagation, by using a **convolution** operation involving the fonction "f" and an appropriate function, which can be called "free propagation operator in vacuum" (always for a monochromatic wave

$$D(\lambda, z) = \frac{\exp(i\frac{2\pi}{\lambda}.z)}{\ldots} \quad \exp(\frac{i.\pi}{\ldots}(x$$

And finally the compacted writing

$$\mathbf{U_z(x,y) = U_0(x,y) \ast D(}$$

Using the convolution formalism will allow making easier some operations on complex amplitudes. What we shall see later.

4. Fourierizing Properties of Thin Lenses

We consider thin lenses to be assimilated a screen of sufficiently low thickness and neglecting as not substancial, the propagation within the screen.
Thus only the optical paths introduced by this screen will be taken into account.

Let us first consider a plate with a parallel face of index "n", placed in front of a transmission pupil Π(ξ,η) , and a plane wave of uniform amplitude 1, meeting the diffracting screen. The glass thickness passed through is "e", the optical thickness passed through is "n.e". The phase variation affecting the wave at the crossing is

$$\Phi(\xi,\eta) = 2\pi\,\lambda.\,n(\xi,\eta).\,e(\xi,\eta)$$

Here we assume a homogeneous index, i.e. independent of the coordinates

$$\Phi(\xi,\eta) = (2\pi/\lambda).\ n.e(\xi,\eta)$$

The introduction of the blade consists of replacing the path $e(\xi,\eta)$ by $n.e(\xi,\eta)$ which brings a resulting phase shift $(n - 1).e(\xi,\eta)$. Thus we work with a screen whose transmission is given by the factor

$$tL(\xi,\eta) = \exp[\ i\ (2\pi/\lambda).\ (n - 1).e(\xi,\eta)\ ]$$

We can write this transmission $\exp[\ i\ 2\pi\ \lambda\ .h(\xi,\eta)\ ]$ by referring to an effective thickness h, which takes into account the optical index of the glass.

The manifestation of $\exp(\ i\ 2\pi\ (n - 1)\ .e/\ \lambda\ )$ will be one more global factor, coming out of the integral and which will disappear in calculating the intensity.
In short, if our glass slide is a convex plane lens of radius of curvature z.(n - 1), we will have in the plane (x,y) located at distance z from the diffracting screen, a complex amplitude described by:

If $U(\xi,\eta)$ represents the complex amplitude at the entrance of the screen, the complex amplitude in the observation plane at the distance z will be

$$U(x,y) = \frac{\exp(i\frac{2\pi}{\lambda}.z)\ \exp(i\frac{\pi}{\lambda z}(x^2+y^2))}{i.\lambda.z} \cdot \int^{+\infty} \Pi(\xi;\eta).t_L(\xi;\eta).\exp(\frac{i.\pi}{\lambda.z}(\xi^2+\eta^2)).\mathbf{exp}(-\frac{\mathbf{i.2\pi}}{\boldsymbol{\lambda.z}}(\mathbf{x\xi+y\eta}))\ d\xi.$$

which is also written:

$$U(x,y) = \frac{\exp(i\frac{2\pi}{\lambda}.z)\ \exp(i\frac{\pi}{\lambda z}(x^2+y^2))}{i.\lambda.z} \cdot \int^{+\infty} \Pi(\xi;\eta).\exp(i.\frac{2\pi}{\lambda}.(\frac{(\xi^2+\eta^2)}{2z}+h(\xi;\eta))).\mathbf{exp}(-\frac{\mathbf{i.2\pi}}{\boldsymbol{\lambda.z}}(\mathbf{x\xi+y}$$

If we know how to make a slide such that

$$\exp(i.\frac{2\pi}{\lambda}.(\frac{(\xi^2+\eta^2)}{2z}+h(\xi,\eta$$

we will calculate U(x,y) by the TF of the amplitude $\Pi(\xi,\eta)$ at the entry of the plate and no longer that of $\Pi(\xi,\eta).\ \exp(\ i.\pi\ \lambda.z\ (\xi^2+\eta2)\ )$, what is greatly more pleasant.

The problem therefore amounts to manufacturing a thickness of glass $e(\xi,\eta)$ such that

$$\frac{2\pi}{\lambda}.\left[\frac{(\xi^2+\eta^2)}{2.z} + (n - 1).e(\xi,\eta)\right] = 0$$

C'est-à-dire $e(\xi,\eta) = -\frac{1}{n-1}.\frac{(\xi^2+\eta^2)}{2.z}$. **Une épaisseur de verre négativ**

A negative thickness of glass !!! (???) !!!

Yes but negative for its part depending on (ξ,η), in fact this means that we must have a thickness $e(\xi,\eta) = e - (\xi^2 + \eta^2)(2.z.(n - 1))$, where “e” is the thickness at ($\xi = 0,\eta = 0$) .

In short, if our glass slide is a convex plane lens of radius of curvature z.(n - 1), we will have in the plane (x,y) located at distance z from the diffracting screen, a complex amplitude described by:

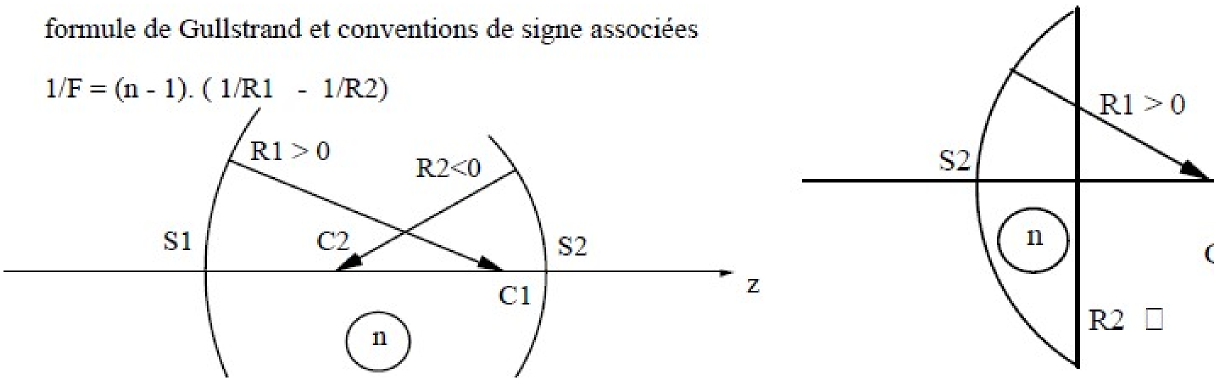
$$U_z(x,y) = \frac{\exp(i\frac{2\pi}{\lambda}.z)\exp(i\frac{\pi}{\lambda z}(x^2+y^2))}{i.\lambda.z}.$$

$$\int \Pi(\xi;\eta)\exp(\frac{i.\pi}{\ldots}(\xi^2+\eta^2))\exp(i\frac{2\pi}{\ldots}(n-1)\frac{(\xi^2+\eta^2)}{\ldots})\exp(\frac{i.2\pi}{\ldots}(x$$

or more simply (hum ??):

$$U_z(x,y) = \frac{\exp(i\frac{2\pi}{\lambda}.z)\exp(i\frac{\pi}{\lambda z}(x^2+y^2))}{i.\lambda.z}.\int \Pi(\xi;\eta).\mathbf{exp}(-\frac{\mathbf{i.2\pi}}{\boldsymbol{\lambda.z}}(\mathbf{x\xi} +$$

The focal length of a plano-convex lens (radii of curvature R1 = R and R2 = ∞) is given by:
1/F = (n - 1). (1/R1- 1 /R2) = (n - 1). 1/R   Gullstrand formula, see below



Such a screen has a transmission (note that « n » here is the optical index of the glass, not equal to 1)

$$L'(\xi,\eta) = \exp\left(i\left(\frac{2\pi}{\lambda}.\left[e - (n-1).\frac{(\xi^2+\eta^2)}{2R}\right]\right)\right) = \exp(i\frac{2\pi}{\lambda}.e).\exp[-i.(n$$

or again (by forgetting the constant phase factor)

$$L(\xi,\eta) = \exp (- i\pi (\xi 2 + \eta 2) / (\lambda.z)).$$

This is the expression of the transmission of a convex lens with focal length “z)
We will therefore have for the amplitude in the focal plane (here “z” is replaced by the focal length “F”)

$$U_F(x,y) =$$

$$\frac{\exp(i\frac{2\pi}{\lambda}.F).\exp(i\,\frac{\pi}{\lambda F}(x^2+y^2))}{i.\lambda.F} . \int \Pi(\xi;\eta).[\exp(\frac{i.\pi}{\lambda.F}.(\xi^2+\eta^2)). L(\xi;\eta)].\mathbf{exp}(- \frac{\mathbf{i.2}}{\mathbf{\lambda.}}$$

which ultimately (the factor in square brackets is now 1) is reduced to:

$$U_F(x,y) = \frac{\exp(i\frac{2\pi}{\lambda}.F) \exp(i\,\frac{\pi}{\lambda F}(x^2 + y^2))}{i.\lambda.F} . \int \Pi(\xi;\eta).\mathbf{exp}(- \frac{\mathbf{i.2\pi}}{\mathbf{\lambda.F}}\mathbf{(x\xi} +$$

$$= \frac{\exp(i\frac{2\pi}{\lambda}.F) \exp(i\,\frac{\pi}{\lambda F}(x^2 + y^2))}{} \; [\hat{\Pi}(u;v)] \quad u = \frac{x}{\lambda.F}$$

Thus, the addition of a thin lens in the diffracting plane makes it possible to obtain in the focal plane a distribution of amplitude U(x,y) calculable by the TF of the complex amplitude reaching the lens. In this FT we assign respectively the values x/ λF and y/ λF to the conjugate variables u and v.

Isn’t that great ??!!
Let's recap:
If I know the complex amplitude distribution in a frontal plane (i.e. perpendicular to the direction of propagation considered) placed at position "z_1"
Then I can calculate the complex amplitude distribution in a frontal plane at position "z_2".
The tool for this work is called the **Fresnel transformation**.

If at"z_1" I place a thin converging lens, of focal length ($F=z_2 - z_1$) then the tool for the same job is simply a Fourier transform.
And there you have it, (almost) All of Fourier's optics fits in there. Obviously we must not forget the conditions needed for this formalism:

scalar approximation
Fresnel conditions
paraxial conditions

end of chapter 1